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Observations on the Study and Teaching of Mathematics*

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I have chosen for the subject of my discussion this evening the rather broad topic "The Study and the Teaching of Mathematics." Doubtless some of you will think that either of these subjects is too broad to be covered in one evening's discussion, and that I should at least confine myself to one of them. But the two things are so closely intertwined that I do not wish to separate them. The study and teaching of mathematics go hand in hand. Most of us who study mathematics do so under the direction and guidance of some more experienced person who has been along that way himself and who is, or should be, fully acquainted, not only with the pitfalls and the difficult spots in the path, but with the beautiful scenery on all sides which we can enjoy provided only it be pointed out to us. It is true that there are notable exceptions to this rule, the best known of which is perhaps that of Blaise Pascal. But not many of us are Pascals. In our study of mathematics the difficulties in our path would be so great and we would be so busily engaged in getting over them that we would scarcely have the time, even if we had the ability, to see the beauties that lie all around us. Hence we need a guide. For that reason I have had the temerity to include both the study and teaching of mathematics in the title of this discussion tonight.

The study of mathematics is by its very nature different from many other studies. Not because of the symbolism employed, for that is a superficial aspect of the subject. In fact, for many hundreds of years mathematics was not symbolic but rhetorical; that is, all statements were written out in full, not by means of a compact symbolism. Indeed, the symbolic mathematics of today is largely the development of the last four or five centuries. But the difference is more fundamental than that. Mathematics is a *structure*. Each proposition or theorem is based on postulates, propositions and theorems that come before, just as the first floor of a house is built up on the foundations, and the second story is built up on the first; and so on. In most cases it would be impossible for one to pick up a book on mathematics and read understandingly page 101 without having read previously the first hundred pages. Moreover, just as the mortar and the cement in the foundation should be allowed to harden before the first floor is added on, so the student should learn a little mathematics every day and assimilate that, or let it *set* as concrete does, before adding more mathematics. Thus,

* An address delivered before the Pi Mu Epsilon Mathematical Fraternity at Duke University, March 11, 1936.

Left to right, top row:—William Betz, H. C. Christofferson, M. L. Herring, Mary Potter, Woolsey, J. T. Johnson, E. R. Breslich. Bottom row—Mary Kelly, Florence Brooks Miller, Martha Hildebrandt, W. S. Schlauch, Vera Sanford.

by its very nature mathematics is a discipline that requires unflagging daily attention.

One cannot read mathematics as he would read a novel, although to be sure some students try to do so. When I was in college, I heard of a student who studied mathematics in such an unusual manner that I must describe it to you. After a hearty dinner this student would return to his room in the evening and make detailed preparations for his task. He would first pull his big Morris chair over to one side of the fire place. He would then bring up his smoking stand and place it close by. Next, he would take off his coat, slip on his lounging robe, pull a big eye shade down over his eyes get out his big calabash pipe and then sit down in the chair. One final step and he would be ready for hard work. This consisted in removing his shoes and propping his feet up against the wall. And in order that his feet would not slide down, this ingenious young man had his bedroom slippers tacked to the wall! He thought he was now ready for work. But little work he did as you can well imagine. He was too comfortable to cope with the difficulties that he encountered.

Many instructors realize that there are difficulties in the study of mathematics, and they are ready and willing to aid their students over the rough spots. They go into great detail, explaining step-by-step, so that finally when the student comes to the end, he knows and can follow every step in the proof, but he does not know what it is that he has proved! In other words, he cannot see the forest for the trees. I believe that this situation is more common than some of us realize. Even as a first year graduate student, I came to a realization of the fact that I was doing just that. For that reason, I now tell my students that it is far more important to know what it is they have proved and the significance of it than it is merely to understand the proof step by step. Of course, it is desirable to do both, but if a choice is to be made, I believe the former to be the more important.

Then comes the question of *memory*. Most students have had it drummed into them from high school days that one should not memorize. It is true that one should not memorize pages of proof. But a good memory is indispensable, nevertheless, and when one has learned a theorem, of what use is it to him if he immediately forgets it? Mathematics is by its nature a structure, and how can one erect a structure if the substructure crumbles as fast as it is built? How can one use a theorem that he has proved in the proof of others if he immediately forgets the theorem?

In 1920 I sat in on a course of lectures on *General Analysis* under the late E. H. Moore of Chicago. One morning he asked his class to name three important attributes that any mathematician should possess. Most of us replied in one way or another and then Professor Moore gave his answer to the question. The three attributes that he gave were *memory*, *coherence* and *correctness* in that order. By *coherence* he meant the ability to make an *intelligible* statement, which he said was more important than making a *correct* statement. So you see that *correctness*, which all of us had put in first place, he placed third, while *memory*, which we had all neglected, he put in first place.

And now I turn to the question of the teaching of mathematics. As I have said, the instructor is a guide who should be able not merely to point out to his students the difficulties in their path and to assist in surmounting them, but also to point out the implications of the mathematics they have studied with its applications to daily life, and its interrelations with other branches of mathematics. Nearly all instructors can, in a measure at least, fulfill the former condition, but relatively few, I believe, can fulfill the latter satisfactorily. Mathematics need not be made an uninteresting subject. It is not at all necessary that our students

go out of our classes, as so many of them do, with their only recollection of the course being "For Tuesday we will work ten problems on page 153." I believe this is an important matter epecially since there has been such a concerted movement in some quarters to abolish mathematics as a requirement.

It has been said by some specialist in the field of Education that "a *good* teacher does not have to teach his students; all he needs to do is to guide them as they learn." This may be true in some subjects (I have serious doubts); but from my own experience and observation, I am convinced that it is not true in mathematics. If the students are enthusiastic about their subject and are anxious to learn (as, unfortunately, many are not) it is true that a poorly informed but *good* teacher may guide his students. However, the students may be guided incorrectly. Without the teacher's being aware of what is happening, they may be led into fallacious reasoning and into incorrect conclusions, from which they never recover.

And so I list as a first and fundamental requisite of a good teacher a thorough knowledge of his subject. One cannot know too much about his subject. I have heard superintendents and principals remark that they did not want to hire such and such a person because he knew too much mathematics to be a good teacher. Such an idea is utterly fallacious. One of the most interesting and inspiring teachers that I ever had was Professor Wilczynski of the University of Chicago, who, as a scholar, was without a peer in his field. We have often heard it said that the greatest men are the simplest, and I think that was eminently true of him. I suppose that at some time or other every one of us has had to listen to a teacher deliver a discourse that was utterly unintelligible, and at the end of the ordeal some one would remark that the lecturer knew too much about his subject. On the contrary, it may well be that he knew too little. The field of mathematics is so vast and the reasoning is sometimes so intricate that it is a comparatively simple matter for one who is not so well informed to read up on an obscure topic and deliver a lecture that will baffle even specialists in the field. So I maintain that the habit of delivering obscure and unintelligible lectures is no indication of great knowledge of the subject; nor is the habit of delivering lucid and clear lectures an indication of lack of acquaintance with the subject.

It is true that knowledge alone does not assure one's being a good teacher. The ability to teach is an art with which some seem to be natively endowed, but which others of us must acquire at great pains. But it is my belief that, with the possible exception of certain isolated cases, this ability can be acquired provided only one makes up his mind to do it. It is true that one can devote so much time to the acquiring of knowledge of his subject that he has little time or energy left for the study of the question as to how best to present it. There is indeed a danger in that, but no one could argue that on this account one should hesitate to learn all he can.

The question now arises as to how one can attain an adequate knowledge of the subject that he is too teach. First of all he should make a thorough study of it in school and college. But that alone is insufficient. Mathematics is a very easy subject to forget, and unless one continues to read and study he will soon lose a great part of what he once knew. I once heard Professor William Cain, head of the Department of Mathematics in the University of North Carolina and perhaps the most eminent mathematician in the state, remark that he had forgotten more mathematics than he knew. And after all, even if we get a good course in college, we really have learned very little mathematics. Any one who relies solely on what he has learned in college can never hope to be a mathematician.

Not every one of us can be a Gauss, a Weierstrass or a Kronecker. Perhaps very

few of us can be Felix Kleins, but it seems to me that we can aim in that direction. I may be mistaken, but my opinion is that Klein has had as profound an influence on mathematics as any man of the past century. Klein was a good research mathematician too, but I leave that out of consideration for the present. He was above all a *teacher, par excellence.* What was the quality that Klein possessed that gave him this great ability? First of all, it was his native genius, of course, but secondarily it was his breadth of knowledge of mathematics. Klein must have been a voracious reader. A mere perusal of his "Elementary Mathematics—From an Advanced Standpoint"[1] is sufficient to convince one of the catholicity of his interests. And if one is to be an inspiring teacher of mathematics, and in particular of elementary mathematics, I believe he must do just as Klein did—*read!* We cannot all be great research men. Many recognize that fact and fall back on the statement "Well, I pride myself on being a good teacher"; but their actions belie their words. Not only do they attempt no research, but they even refuse to read what others have done. As a consequence, they are very poor guides through the intricacies of mathematics. They are able to recognize the stereotyped difficulties which a student meets in his work and can, after a fashion, iron out those difficulties for him. But as for pointing out the great relating principles and throwing in illuminating illustrations that are not in the book, they are unable to do it.

The question naturally arises as to what one should read. The field of mathematics is becoming so vast and the reading of some mathematical papers is such a slow process for most of us that we would become gray headed before making a perceptible start. My answer is that anyone who desires to become a mathematician should read first of all some good History of Mathematics,[2] and if possible he should take a course in the history of the subject. I am aware of the fact that many good mathematicians disagree with me on this point. They hoot at the idea of a good mathematician's wasting his time reading history. But I hold with Glaisher that "no subject loses more than mathematics by an attempt to dissociate it from its history."

Suppose, for example, that in algebra we are studying the solution of the cubic equation. We know that Euclid in his *Data* solved quadratic equations graphically, but that the first cubic equation (except the simple one $x^3=2a^3$) was in the work of Archimedes (230 B.C.) in connection with his investigation of the volumes of segments of spheres. The next cubic encountered was in the writings of Diophantus (3rd century A.D.), after which there was no mention of cubics until about 1000 A.D. The poet Omar Khayyam, the author of the *Rubiyat*, could solve any cubic graphically, and he is said to have been the greatest authority on cubics of his time. He is credited with the statement that he doubted that the general cubic would ever be solved algebraically. Yet, four and one half centuries later, Tartaglia, Cardan, Bombelli and others showed how to solve any cubic by using radicals. The controversy between Tartaglia and Cardan, of which I always give an account in my classes never fails to arouse interest on the part of my students, and I believe I can safely say that although one year later, not one could solve a cubic, practically all will carry away with them the significance of my remarks on the history of it. So I say that anyone who claims to be a mathematician or a teacher of mathematics should acquaint himself with the History of the subject.

Then again, one should read works on the philosophy of mathematics, such as Russell's "Introduction to Mathematical

[1] Klein, Felix, *Elementary Mathematics—From an Advanced Standpoint.* Translated from the third German edition by E. R. Hedrick and C. A. Noble. The Macmillan Company, 1932.

[2] Any good history will do such as Ball, Cajori, Smith or Sanford, in English, Marie in French, or Cantor in German.

Philosophy," or his "Principles of Mathematics," Shaw's "Philosophy of Mathematics," and the like, so as to be in a position to answer questions concerning zero, the infinite, etc., that students inevitably ask. J. W. Young's "Fundamental Concepts of Algebra and Geometry" and J. W. A. Young's "Monographs on Modern Mathematics" should be read by all, and in the field of recreations I should recommend Ball, Schubert and White. This should be only a beginning and I cannot attempt to list the books on mathematics that one should read. That will be determined to a large extent by the particular interests of the individual. But the point that I am making is that *one should read.* Even if one is primarily a research man, and I think it highly desirable, and in fact necessary, that a University man perform some research, still my feeling is that after performing several pieces of research, one should lay aside for a time his research projects, take his nose out of the little corner and devote himself to the reading of mathematics in general. My belief is that Felix Klein did just that.

The question of zero is always an intriguing one to students. Although they have known for a long time that the product of every number multiplied by zero is zero and have accepted that fact without question, still when they have occasion for the first time to attempt to divide by zero, then only is their interest aroused. This usually happens when in trigonometry they attempt to find the functions of 90°. On one occasion I had a student working at the black board and when I examined his work I found that he had written

$$\tan 90° = 1/0 = 1.$$

In reply to my question as to why he put $1/0 = 1$, he made the following remark: "When you divide 1 by zero, you are not dividing 1 by anything, and if you don't divide 1 by anything, you have 1 just as you had when you started. So the answer is 1." Of course this is pure sophistry. We know that the statement "We are dividing 1 by 0" is a *positive* statement, while the statement "We are not dividing 1 by anything" is a negative statement and in the latter case we are not saying what we are attempting to do. Hence, we have no clue as to what the result will be.

There is a big distinction between *zero* and *nothing.* Zero is a number, *viz.,* that number which results when 1 is subtracted from 1, or 2 is subtracted from 2, etc. But *nothing* is not a number. Quite recently, I heard *nothing* described as "zero with the rim knocked off." This is somewhat in line with the statement of an old professor of mine. "This" said the professor, as he wrote a cipher on the board, "is zero, while this" as he erased the cipher and pointed to the blank board "is nothing."

A little discussion is sufficient to convince the student that 1/0 yields no quotient, while 0/0 yields as quotient any number whatever. That is, the latter is completely indeterminate and therefore "the only thing the symbol 0/0 is good for is to be rubbed out." Hence, since division by zero never yields a determinate quotient, this operation is excluded from algebra; "First, because it is nowhere necessary, and second, because of the ambiguities and contradictions that it gives rise to" to quote Pierpont. Or, in the words of Wilczynski, "The thirteenth commandment should be 'Thou shalt not divide by zero.' "

Since zero is the only number in the number system by which we cannot divide, and since it is the only number, real or complex, which has no logarithm, it a sort of black sheep of the family, and for that reason some students think it should be denied citizenship in the number system. But zero is useful, nevertheless, in spite of its shortcomings, and we could not get along very well without it.

A story is told of an Israelite, who during a time of famine in Palestine went down to Egypt for the purpose of buying grain. Upon his arrival he found that he

could secure the desired provisions and the steward weighed out for him 48 measures of it. Upon being asked the bill, the steward replied that the price was 27 pieces of money per measure, but that he was unable to multiply so that it would be necessary for the Israelite to wait until he could get the Pharaoh to multiply for him. At that moment the Pharaoh was deep in meditation and, somewhat vexed at being disturbed, he remarked that he was going to teach the steward how to multiply so that henceforth he need not be disturbed. The directions were to proceed as follows:

Place the 27 and the 48 in parallel columns. Under the 27 place 13, its quotient by 2, the remainder being disregarded. Under the 48 place its double 96. Similarly, under the 13 place 6, its quotient by 2, the remainder being again disregarded, while under the 96 we place its double 192. This process is continued until in the 27 column a quotient of 1 is obtained. From the 48 column delete those numbers, here 192, which corresponds to even numbers, here 6, in the 27 column. The sum of the remaining numbers in the 48 column will then be the product of 27 by 48.

27	48	48×1
13	96	48×2
6	~~192~~	48×2^2
3	384	48×2^3
1	768	48×2^4
	1296 = 27×48	

Thus, the process of multiplication is replaced by repeated halving and doubling, a process which could be performed on the fingers, if necessary. This scheme is said to have been employed until fairly recently by certain peasant tribes in Russia.

At first glance it seems quite remarkable that this process should work, especially since in the halving the remainders are always neglected. But it will always work, whatever be the two positive integers to be multiplied, and the explanation is very simple. It is based on the easily established proposition that every positive integer can be expressed as a sum of powers of 2. Thus,

$$27 = 2^4 + 2^3 + 2 + 1,$$

so that

$$27\times48 = 2^4\times48 + 2^3\times48 + 2\times48 + 48.$$

But on reference to the scheme above, it will be seen that this is precisely the sum that we have formed, the product $2^2\times48$ being missing since the term 2^2 does not enter the expression for 27.

This is a very simple problem, but it seldom fails to elicit interest from a class and it is remarkable how few young teachers of mathematics are acquainted with it.

Along somewhat the same line is the following:

A certain merchant had an object that weighed exactly 40 pounds. One day the object dropped to the floor and broke into four pieces. Upon examination the merchant found that by using these pieces suitably in one or both pans of his balances, he could weigh any integral number of pounds from 1 to 40. Find the weights of the four pieces.

This very interesting problem was given me by a mathematics teacher in of our high schools. As soon as I began to work on it, I recognized it as not belonging to the ordinary run of "barber shop" problems, and it did not surprise me much to find out later that it was a classical problem considered by the celebrated French Diophantist, Bachet de Meriziac (1581–1638). It is now known as Bachet's Problem of the Weights, although it may have been considered earlier by Tartaglia (1536).

In considering this problem, let us denote the weights of the four pieces by x, y, z and w, respectively. We then have the condition:

$$x + y + z + w = 40,$$

and, in so far as expressing the problem in algebraic equations is concerned, this seems to be the only equation that we

can get. In fact, we do not know as yet that x, y, z and w must be integers. We, therefore, seem to have a Diophantine problem of the worst kind. However, let us inquire as to how many weights we can get by shifting the pieces from one pan of the balances to the other.

By using the pieces one at a time we can get $4 = {}_4C_1$ weights of the form x, y, etc. By using the pieces two at a time, we can get ${}_4C_2$ combinations, but since the two pieces can be placed either in the same pan or in different pans, this number must be doubled. Hence, by using two pieces, we can get $2 \cdot {}_4C_2$ weights. These are of the form $x \pm y$, $x \pm z$, $x \pm w$, etc. Similarly, by using three pieces we can get $2^2 \cdot {}_4C_3$ weights of the form $x \pm y \pm z$, $x \pm y \pm w$, etc., and by using all four pieces we can get $2^3 \cdot {}_4C_4$ weights of the form $x \pm y \pm z \pm w$. Hence, the total number of weights that can be gotten, by placing the pieces in the same pan or in different pans, is

$$n = {}_4C_1 + 2 \cdot {}_4C_2 + 2^2 \cdot {}_4C_3 + 2^3 \cdot {}_4C_4 .$$

But by the binomial theorem:

$$3^4 = (1+2)^4 = 1 + 2 \cdot {}_4C_1 + 2^2 \cdot {}_4C_2 + 2^3 \cdot {}_4C_3 + 2^4 \cdot {}_4C_4,$$

whence,

$$n = (3^4 - 1)/2 = 40.$$

It now follows at once that if we are to get 40 different integral weights out of the four pieces, the pieces must all be of different weight, and each must weigh an integral number of pounds.

If x, y, z and w be the weights of the pieces in descending order of magnitude, it will be seen on reflection that the largest weight obtainable is through the combination $x+y+z+w=40$, while the next largest is by the combination $x+y+z$, and since by the conditions of the problem this must equal 39 pounds, it follows that $w=1$. We can then conclude after some argument that the weights of the four pieces are

$$x=27,\ y=9,\ z=3,\ w=1,$$

and that the solution is unique. It is seen that these numbers are consecutive powers of 3; and one might guess at once that the explanation of the problem lies in the simple fact that "every positive integer can be expressed as an aggregate of different powers of 3, the terms in the aggregate being combined by the signs + or − appropriately chosen" (Carmichael, *Theory of Numbers*, Ex. 1, p. 24). Thus,

$$16 = 3^3 - 3^2 - 3 + 1. = x - y - z + w.$$

Hence, if the merchant desired to obtain a weight of 16 pounds he would proceed as follows: Place the x and w weights in one pan of the balances. In the other place the y and z weights. If then sufficient material be added to the latter pan so as to get a balance, the weight of the material would be exactly 16 pounds.

As a final illustration, let us consider a problem that appeared in a popular magazine a few years ago. Write down the number

$$x = 142857.$$

On the forming the products $2x$, $3x$, $\cdots$, $6x$, it will appear that the digits in each of the products are precisely the digits in the number x, and in the same order, but cyclicly interchanged. Thus

$$2x = 285714,\ 3x = 428571, \text{ etc.}$$

On the other hand

$$7x = 999999.$$

What is the explanation of this strange phenomenon? The fact that $7x = 999999$ will give us a clue, *viz.*, that the digits in x are the digits in the repetend of the fraction

$$1/7 = .142857\ \ 142857 \cdots.$$

But not all repetends have this property, for example

$$1/13 = .076923\ \ 076923 \cdots$$

and if we take

$$y = 076923$$

then

$$2y = 153846,$$

so that y does not possess the remarkable property that x does. The explanation lies in the fact that the fraction 1/7 yields a

perfect repetend, while 1/13 does not. That is, 7 is a prime p such that the repetend of the fraction $1/p$ contains exactly $p-1$ digits. The same is true of the number

$$x = 0588235294117647$$

composed of the digits of the repetend of 1/17. Thus,

$$2x = 1176470588235294; \text{ etc.}$$

These simple illustrations serve to show the kind of thing that I am thinking of. There are many, many others that a teacher can get from his reading and which will serve to arouse and stimulate the interest of the student. It was just this kind of thing that Felix Klein could do, although he could do it so much better—in his own inimitable way.

In conclusion may I say that not every one of us can be a Newton or a Gauss, and it is well that we cannot be. But if we do not devote our lives to the discovery of truth, we can at least *read* that truth which others have discovered. We can assimilate those truths and make them ours so that in our teaching we may not merely serve for smoothing over the difficulties that our students encounter, but we can act as guides and point out to our students the interesting and beautiful scenery that they would otherwise miss as they go along. In other words, we can follow as our example that inimitable master, Felix Klein.

WE ARE all concerned in courses of studies, departmental officials not less than school administrators and teachers. On this continent perhaps more than anywhere else educators have swung away from tradition. We are living in a utilitarian age, and naturally a good deal of emphasis is being placed on a practical education, one which fits the youth of the nation for vocations and for citizenship. Learning for learning's sake is no longer the slogan of the schools. There is, however, a real danger in too great a swing away from the hard disciplinary subjects of study. I am thinking mainly of secondary school curricula. The elementary schools do not present to the same extent a problem in this respect. They are concerned mainly with developing skills in using the tools of learning. Instead, however, of labouring this question of curricula, may I read you an extract from an address given by the President of Harvard University at its recent tercentenary celebration. Dr. Conant says:

> The older educational discipline was based on the study of the classics and mathematics: it provided a common background which steadied the thinking of all educated men. We can not bring back this system if we would, but we must find its modern equivalent. Like our ancestors we must study the past, for "he who is ignorant of what occurred before he was born is always a child." In my opinion it is primarily the past development of our modern era which we must study and study most exhaustively and critically. We must examine the immediate origins of our political, economic and cultural life and then work backwards. We must now, however, spread the inquiry over so wide a range that the average man will obtain only a superficial knowledge. It does not seem to me to be a step in the right direction to dip our children first in one barrel of tinted whitewash and then in another. The equivalent of the old classical discipline is not to be found in a bowing acquaintance with universal history and general science and an exposure to scattered examples of art and literature. Our present educational practice, which insists on the thorough study of at least one discipline, is certainly sound.

Rogers, George F., "President's Address, Canadian Educational Association," *The School*, January, 1937, p. 384.

Mathematics and the Integrated Program in Secondary Schools

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In the *Journal of the National Education Association* for May, 1934, there appeared on page 137 the following comment on the "trend toward integration" in American secondary schools:

> The isolated subjectmatter compartment type of curriculum is rapidly losing ground. In its place is being substituted an integrated (correlated, fused, articulated) curriculum in which the plan of work for the pupil cuts across subjectmatter lines. The primary objective is the substitution of "pupil goals" for "subjectmatter goals." In achieving the former all subjectmatter fields which have a contribution to make are called upon. This is a momentous shift of emphasis. No more significant change has even been attempted in the secondary schools.
>
> Integration solves one set of problems but raises others which are vitally important. It completely changes the pattern of secondary school instruction. A major problem becomes the determination of the goals of instruction and learning. Teaching in the various subjectmatter fields in the past, with all its weaknesses, was at least a fairly safe procedure; the results sought were definitely known. Under the integration plan, subject matter as such becomes a means to an end, the goal being an understanding, an appreciation, or a skill. The question of objectives of learning becomes a paramount issue.
>
> A second difficulty in making this adjustment grows out of the fact that the faculty is composed largely of specialists. The principal is a specialist in administration, and too often his time is largely absorbed with administrative detail. Each teacher is a specialist in one or two subjects. *There is need for more people who will make themselves masters of secondary education as a whole,* not only from the organization standpoint, but from the point of view of the total content of its teaching. H. G. Wells has made an attempt at such integration in his *Outline of History, The Science of Life,* and *The Work, Wealth and Happiness of Mankind.*

There is no doubt that the failure to heed such caution will lead to a situation as bad as, or worse, than the one which the "integrated program" is intended to correct. However, there are other reasons why the "trend toward integration" may result in disaster unless care is exercised.

In the first place, we ought to come to some agreement as to what "integration" means. Funk and Wagnall's *Practical Standard Dictionary* (p. 601) defines "integration" as "The act or operation of integrating; the bringing together of parts into a whole." Many people use the terms "integration" and "correlation" interchangeably. I believe this is unfortunate. A few decades ago "correlation" was the rallying cry for reform in teaching—the use of one type of subject matter to reinforce and clarify another. Ideal or absolute correlation means blending all courses into one single course.

In attempting to correlate content material, subjectmatter teachers, and particularly administrators, have made the mistake of trying to correlate by merely changing the names of courses, calling algebra and geometry "correlated mathematics," history and geography "social studies," and the like. If subject matter is correlated, it must be properly articulated for teaching purposes. For example, the various parts of mathematics may be correlated with each other, or mathematics may be correlated with the other great fields of knowledge, such as science, art or music. In the ideal situation we must do both. Integration, on the other hand, is a psychological term and refers to what goes on inside the pupil. An ideal program of integration therefore accepts that subjectmatter which harmonizes with the life ex-

periences of the pupil. It is our task as teachers to see that only those content materials are chosen which will best fit the pupil's needs (using needs in the broad sense).

According to Professor Caldwell

> The program of studies of the high school, like its predecessor in the college, has grown up as a series of separate subject interests. Most high school programs seem to indicate either that we believe in dissociated subjects, or that we are helpless in finding and using interrelations. It is not an integrated program that we offer to the high school pupil. Out of the confusing display of subjects, the pupil is supposed to accumulate sixteen units of credit of which the language requirement is the chief specified, and as yet the most coherent part. He may choose different lines or work, or in many schools he may choose the individual subjects year by year. If the separate choices are to result in an organized education for him, he or someone should coördinate these separate studies. This is not commonly done for him by his high school, and not commonly done by himself. Does this mean that there are not centralizing aspects either in pupils' needs or in the subjects studied? Does it mean that the best we can do is to teach those divisions of knowledge, leaving it entirely to later chance occasions to bring together the related elements of our subjects? We are providing the pupil well-constructed building materials, all well distributed throughout his intellectual premises. But we do not give him any coördinating blue prints to help him construct his personal intellectual home. Indeed, we know too little, as yet, about individual needs and thus devote our efforts largely to presenting the materials from which we blindly hope that individual needs may be met.[1]

Some of our leading educators and scholars are not certain that the integration movement will furnish the best procedure. Professor Bagley recently said:

> . . . instances could be cited of the breakdown of the integration theory in practice. A widely heralded experimental college, based upon the integrationist doctrine, was abandoned, it is said, largely because the students discovered that they were missing certain learnings that their fellows in systematic courses were getting. This and other experiences, however, do not mean that integrationists have nothing of value to offer. There is a clear place for efforts to relate the learnings in different subjectmatter fields to one another, and to the problems of human life viewed on the largest possible scale; but both theoretical and practical considerations suggest very strongly that, with certain exceptions on the lower levels of learning, integrative learnings are most effective when there is something to integrate.[2]

Carleton Washburne has recently sounded a note of warning and advice to the devotees of "integration." After pointing out the possibly salutary effects of the reaction against the traditional breaking up of the curriculum into subjects set off in water-tight compartments unrelated to each other and unrelated to the life of the child, he says that "Like most reactions, this one has swung to the opposite extreme." He then points out some of the absurd things that he knows have been done in the name of "centers of interest," "activities," "projects," "integrations" and the like, and goes on to say that "A complete interrelation of all things a child studies is unimportant and unnecessary" and that "We find no disintegration of our lives as a result of our divergent and diverse activities." He maintains that

> What is really important is that each thing we teach the child be integrated with his own life and his experience. It must fulfill some need; it must give expression to some impulse; it must contribute to some thinking. It must, in short, have significance. But the significance of one thing may be quite different from that of another, and no harm whatever is done by variety of experience, provided that the separation is between one

[1] Caldwell, Otis W., "The Problem of Integrating High School Studies," *Secondary Education*, Vol. 3, p. 152, September, 1934.

[2] Bagley, W. C., "How Shall We View Elementary Education as Regards: (1) Discipline; (2) The Psychology of Learning; (3) Subjectmatter?" *The Mathematics Teacher*, Vol. 27, pp. 167–81, March, 1935.

subject and another, and not a separation between the subject and the child.[3]

Other important points made by Washburne are that "In mastering a skill, readiness and ripeness are necessary" and that "In mastering a skill certain technics need practice, and the amount of practice needed varies with individuals"[4] and finally that

In mastering knowledge, a certain order and organization are needed. So, too, are certain foundations. To give a bit of knowledge here and a bit of knowledge there, because they happen to be related to a project or a center of interest, is to fail in the orderly development that characterizes the thinking of an educated person.

It is manifestly absurd to suppose it contrary to sound education to be systematic and orderly, to suppose that thoroughness is the antithesis of good learning. Yet the time has come when a person almost has to apologize for a kind of education that develops a subject in an orderly, systematic manner.

The answer does not lie in a reversion to the old compartmentalization of the curriculum, but neither does it lie in an attempt at complete correlation and the kind of "integration" which assumes that all subjects must be integrated with each other and with some center of interest or grow out of some one activity.[5]

If a man like Washburne who is known to represent the so-called progressive point of view will say such things, there must be reason for believing that some of the "progressives" themselves have begun to see the dangers that lurk in panaceas.

I think that the entire situation is admirably summed up by Washburne in the last two paragraphs of his article where he says:

The solution lies in having a basic course required of each child as he reaches the right stage of development, and including in that course only those items which really function or can be made to function in the experience and training of the child. Each of these things—call them subjects if you wish—should be taught in relation to the child's life and interests. They should be taught when the child is ready to use them through having his interest aroused and when he has reached that mental age found by research to be most suitable to the learning of a given topic. And in doing this, the school may well use a number of the old categories—arithmetic is, after all, quite different from social science, and spelling it not related to creativeness and initiative.

On the other hand, any balanced educational program must provide a rich background of experience and activities. It must provide, through electives and through opportunities in connection with common enterprises or activities, a chance for each child to find and follow his own special interests and abilities; a chance for him to use originality, creativeness, and initiative; and most decidedly opportunities for him to work cooperatively with his fellows and to develop a sound social consciousness. These are basic parts of his education. But so, too, is the mastery of those fundamental skills and that body of common knowledge which are essential to living in an organized society.[6]

Speaking of integration, Professor Lindquist says

It is the vogue to build integrated courses. That activity is by no means all loss, for it does tend to break down narrow subject-matter compartments. It enriches greatly each of the various subject areas. Through such cooperative activity the science teacher comes to see that scientific phenomena have social implications and art and language and health implications, and similarly for other specialists. But we do risk the danger of assuming that, because the experiences of the teacher are thus enlarged, the experiences of the children are therefore integrated. In addition to the above it is therefore necessary for teachers to continue to cooperate after the child has had the separate experiences, to the end that these are integrated in terms of more comprehensive problems and more inclusive purposes.[7]

[3] Washburne, Carleton, "The Case for Subjects in the Curriculum," *The Journal of the National Education Association of the United States*, Vol. 26, January, 1937, p. 5.

[4] Ibid.

[5] Ibid.

[6] Ibid.

[7] Lindquist, R. D., "Essentials for a Secondary School," Chapter 3 of *A Challenge to Secondary Education* by Everett and others, p. 139, D. Appleton-Century Co., 1935.

Professor Horn thinks that we do not need new organizations of knowledge. He says

A good many of the exponents of integrated programs seem to assume that we need a new organization of knowledge in terms of life values, that such an organization is an accomplished fact, that current integrated programs exemplify this organization, that they "integrate the child's personality," and that traditional subjects must go. The first assumption is true in the sense that social re-evaluation should be perennial. The other assumptions are largely or wholly erroneous.

That the content and organization of each school subject, as well as the content and organization of the course of study as a whole, should be constantly adjusted to the needs of modern life is beyond debate. Such adjustments must be made, however, by those who know both the subjects and the needs of modern life. This revitalization of school subjects and the enrichment of the curriculum as a whole have been the chief concern of a large number of competent workers for the last twenty-five years. Within the memory of the members of this audience the subjects of the curriculum have been revolutionized in their content, in their patterns, and in the methods by which they are taught. This is true not only of such so-called content subjects as hygiene and history, but also of the so-called three r's—reading, writing, language, spelling and arithmetic. The potentialities of research in the improvement of curricula have not, however, even been approximated.

Proponents of non-subject programs generally advance the argument that such programs, as compared with subject curricula, bear more closely on the present needs of children. What are the present needs of children? The fact is that we do not have in print any respectable pattern or series of patterns of what constitutes a good life for a child of six, of ten, or of any age. Neither the scientific nor the critical analysis of children's needs and interests has advanced very far. Only a few small areas have been investigated, such as the vocabularies of children, their health, their interests in poetry, and certain of their social adjustments. And even if these narrow fields of investigation be pieced out by critical thought, the resulting patterns bear little resemblance to the patterns of typical unit or activity curricula. Indeed, it is generally much easier to show the relationship of subjects to child needs than it is to show the relationship of most non-subject units to child needs. It is clear, for example, that the child's present life is enormously facilitated when he has learned to read and write, but it is not clear that it is equally facilitated by such so-called activities as have become traditional in non-subject curricula.

. . . . . .

I believe that proper integration offers real, although partial, solution to this problem of the cut-up program. For example, the skills and knowledges which are now developed in the three r's can be *motivated*, *practiced*, and *maintained* to a considerable degree in connection with other subjects or units in the school curriculum. In fact, these subjects cannot be taught efficiently out of relation to other subjects or activities. This does not mean that the subjects are to lose their identity or that they are to have no place in the weekly program. It does mean that the time allotments for all the three r's may be greatly reduced and that no one of these tool subjects needs to be taught every day. It is probable, for example, that sufficiently serviceable handwriting could be developed in one thirty-minute period a week. I suspect that the total combined time allotment for arithmetic, spelling, handwriting, the mechanics of composition, and the development of skill and ability in reading could be reduced to not more than ninety minutes a day, providing, first, that these subjects are integrated as needed with other subjects and activities in the curriculum, and second, that the teaching in the special periods devoted specifically to these subjects is guided by the best modern knowledge of learning. The present average weekly time allotment for reading in grade two is over four hundred minutes. I believe that reading would be improved if this allotment were reduced at least by half, and the time thus saved were given to larger opportunities for reading in connection with the other subjects or activities of the school. Heavy reductions could also be made with profit, assuming proper integration, in the allotments in arithmetic, language, writing, and spelling.

This reduction in time allotment for the three r's is justified if the time thus saved is assigned to geography, science, music, health, art, and literature. The values in

these subjects are fundamental, permanent, and universal, and hence suited to the development of the abilities which are integrated with them. But no reduction in the time allotment for these skill subjects should be made if the time which is taken from them is to be used in carving the Tower of Pisa in soap; in learning to imitate farm animals; in making friezes which distort meaning by faulty symbolism; in carving fire boats out of wood; or in historical dramatization which is unauthentic in fact and in spirit. Such enterprises, all taken from modern "activities" or "units," are not only petty in themselves but frustrating in their effect on skills, knowledges, and attitudes which are integrated with them. They are incomparably inferior in worthy purpose, in capacity to stimulate thought, and in their concomitant effects to even the most traditional and formal courses of study in the fundamental skills.

One of the commonest misconceptions concerning integration is that we can't have too much of it. As a matter of fact too much integration is as bad as too little. Clear thinking and effective learning require that the mind focus upon some problem, the knowledge, skills, and attitudes which are pertinent to it, and that it reject all other matters. Multiplicity of association unaccompanied by selection and emphasis, is a deadly enemy of clear thought.

We should integrate, then, not when we can but when we must. This means that the knowledge, skills, and attitudes which are integrated with the central problem must be indispensable to its proper solution. Any deviation from this principle is destructive both to vigorous thought about the central problem itself and to an intelligent use of the integrated subject matter. Thus one of necessity selects certain aspects of the history of Cuba in the study of the geography of that country not as a device for correlating or integrating history and geography but because the geography is otherwise unintelligible.

I am impressed particularly by the frequency with which, in so-called activity programs, the thought of the pupils is distracted from direct and vigorous attack on the central problem by wandering off into extended but petty projects in soap carving, paper cutting, frieze making, and unauthentic dramatization.

The distortion which comes from too much integration is most marked when all or most of the work of the school is organized around *a single theme or problem.* There is no one core of experience to which all else may be subordinated. The course of study, like life itself, must be many-sided.

. . . . . .

There seems to be an assumption in some quarters that it doesn't make any difference what is chosen as a center of integration so long as the pupils like it. Nothing could be more destructive, ultimately, to the movement for integration. The value of any problem, unit, or activity, as the center for integration, tends to vary in direct proportion to its value in child and adult life.

Any integrated programs must rest squarely, therefore, upon a competent analysis of social values. And the more universal, the more crucial, and the more permanent the values inherent in any center of integration, the more satisfactory are the results which accrue to the related subjects or abilities which are integrated with it. It is absurd to subordinate an undeniably valuable skill or body of knowledge to a petty or superficial unit or activity. If the unit of integration be insignificant, the result in the related knowledge, skills, and attiudes will be insignificant also.[8]

On page 57 of the "Preliminary Report of the Science Committee to the Commission on Secondary School Curriculum of the Progressive Education Association," we find the following:

It is thought that the approach to curriculum making illustrated in this report might help to disclose interrelations in the various fields of human knowledge which are of significance in general education. For example, science is seen not as a separate field of knowledge dealing exclusively with the natural environment, but rather as permeating all aspects of living. This broad conception of science leads directly to the relationship of science to the arts, the social studies, language, etc. If this broad conception is applied to all fields, it will tend to break down compartmentalization. The way is thus paved to a more completely integrated program.

[8] Horn, Ernest, "Possibilities and Limitations in Integration," *Curriculum Journal*, Vol. 7, April, 1936, pp. 27–30.

The committee regards the *integration of fields* as a possible device or instrument for aiding the learner to achieve personal integration. Natural centers for integration will be revealed as the generalizations basic to each field are discovered. There is thus provided a basis for developing an integrating curriculum, instead of the familiar practice of "putting subjects together." On the other hand, the generalizations developed in each field will probably reveal materials which on certain levels of pupil development can best be taught in a more specialized context. That is to say, when specialized interests develop, learning activities should take on a more specialized form.

A recent comment by Professor Douglass is pertinent here

In the first place, the significance of much that is and should be taught in schools depends upon the degree to which the learner is able to correlate it with its setting in and application to the outside world. Lacking in experience and orientation in the outside world, even more than his teachers, much of the potential value of his learning is never realized. Even if fairly well understood by children, it is forgotten before they have acquired sufficient acquaintance with its relations to life to enable them to assimilate it and to consolidate their gains. Learning and life, if the former is to be of real value, must be thoroughly interrelated.

As Dr. Judd has so clearly pointed out, the extent to which transfer of things learned to areas of their application and to other fields of learning takes place is conditioned by the ability of the learner to generalize, to carry over the ideas from the situations and conditions in which they are acquired to other situations and conditions. Unfortunately, this power of generalization is not generously bestowed by nature on the great mass of human beings, and it is not wise to build a school program upon any theory which assumes that it is.[9]

No one who follows the trend of the times and who reads widely can fail to appreciate the fact that the static condition of the curriculum is responsible for our failure to make reasonable progress in reorganizing our secondary schools. This condition is due in large measure to the tendency of teachers to cling to the traditional program and also to the fact that they have become so enamored of the subjects they teach that they are not always conscious of what is going on around them. Moreover, they do not appreciate the full significance of recent criticisms of the unsatisfactory situation in our secondary schools.

In this respect mathematics, particularly algebra and geometry, are coming in for an appreciable share of criticism. Everybody from the general educationist to the man in the street feels himself competent to criticize. In four recent plays in New York City, one of the characters in each play takes great satisfaction in criticizing mathematics. One character remarked, "Why, she is as crazy as my geometry teacher," implying of course more than probably was intended. The interesting thing about each incident is that when these attacks were made the audience reacted with a chuckle of approval.

When a group of mathematics teachers was recently reminded of the numerous attacks on the teaching of algebra, one teacher replied, "We should worry, we are no worse than the teachers of English!" Perhaps not. The teaching of English, like that of mathematics and the other subjects, has frequently been bad enough. However, such an attitude should not be consoling to forward-looking teachers. It certainly cannot be of much consolation to the mathematics teachers in Chicago at the present time.

While most of the criticisms of mathematics are hurled at a type of mathematics teaching that has been outmoded in many schools, there is still just cause for complaint. Why is it that so many people who have studied algebra or geometry have such an antipathy for the subject? What is the real reason?

A careful study of the situation in

[9] Douglass, Harl R., "Monograph on Secondary Education," Preliminary Draft for the American Youth Commission of the American Council on Education, 744 Jackson Place, Washington, D. C., April 20, 1936, pp. 48–49.

mathematics does not indicate that the teachers have failed to any more appreciable extent than have the bankers, the economists, the politicians, the captains of industry, or even the schoolmen generally in recent years. However, we know that there is room for improvement in all fields.

The trouble is not so much with mathematics as a subject, but with the stupid way in which it is so often presented to the pupils, even in arithmetic.

It is obvious that a person cannot teach what he does not know, and many of our teachers are not scholarly enough with a sure command of their fields, Or they may know too little outside their own "specialty" to make mathematics properly serve the other great fields of knowledge. What we need more than anything else at the present time is better trained teachers —teacher-scholars, as Professor Bagley calls them, who know not only the subjects they have to teach, but who are also more generally cultured than many of our teachers of formal algebra. We have an oversupply of teachers at present, but we do not have an oversupply of the right kind of mathematics teachers. The day is past when anyone should be permitted to teach algebra who has been trained only to coach athletics. If our teachers were required even to approximate the standards for teaching mathematics that are upheld abroad, we should soon see the dawn of a new day in the teaching of mathematics. What we need is higher standards for teachers. If the educationists will help us here, they can do more good than they are now doing by their destructive criticisms. Cooperation between the educationists and the mathematicians is what we need now.

In the Hall of Science at the recent Century of Progress Exposition in Chicago there was a large mural representing the "Tree of Knowledge," a reproduction of which is shown on page 162.[10]

[10] Copies of this mural suitable for framing or for pasting in the pupil's notebooks (size 6″×9″) may be obtained postpaid from *The Mathematics Teacher*, 525 West 120th St., New York, N. Y. for 5¢ each or 50¢ per dozen. Larger copies for the classroom 25″×38″ in colors may be obtained for 25¢ each postpaid and 20¢ each for each additional copy from the Business Manager of *The Museum of Science and Industry*, Jackson Park, Chicago, Ill.

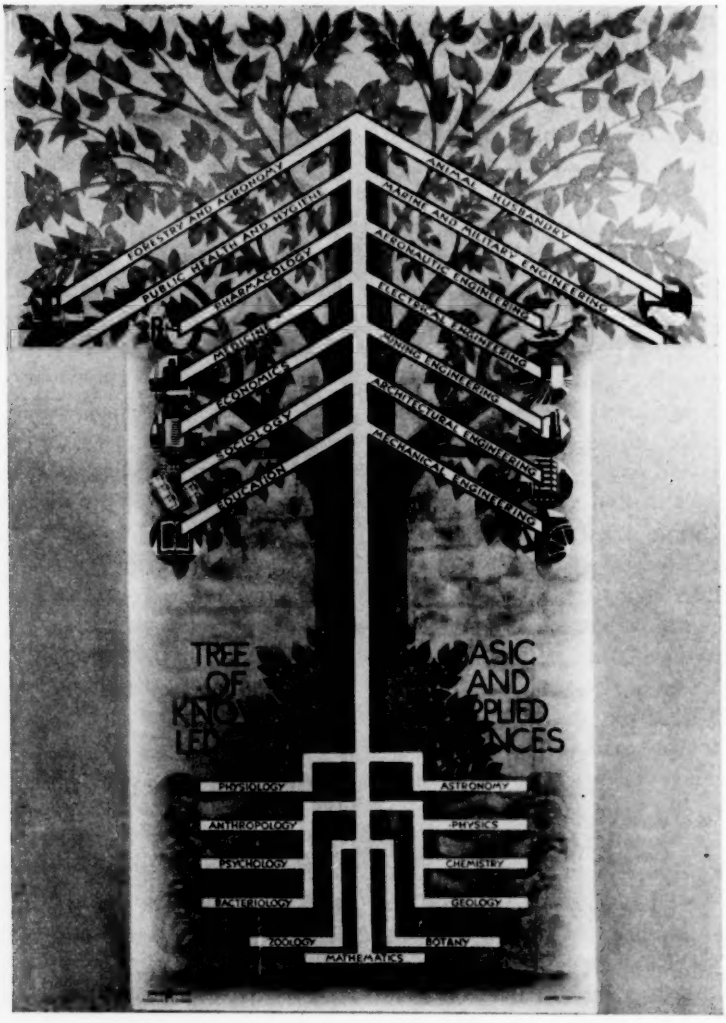
As can be seen from this picture, mathematics is the foundation and the vitalizing energy for the basic sciences such as astronomy, botany, chemistry, geology, and physics and together with them furnishes strong support for the applied sciences in the higher branches, such as the social studies like economics and sociology, and engineering of various kinds.

The fact that mathematics is so important is not surprising to one who is properly informed with respect to the contributions mathematics has made to the other great fields of knowledge and to civilization, but many of our citizens, including a number of educators, are still unaware of the strategic place mathematics really occupies in world affairs today. It should be the business of those of us who are interested primarily in mathematics to help make clear just where and how mathematics can be of real service to the other great branches of learning and to civilization and what can be done to secure these services by a better teaching of mathematics in the schools.

In this complex civilization which we are now entering, a knowledge of mathematics is becoming increasingly important.[11] This does not mean that everyone should be trained to be a mathematician, but it does mean that every well-educated citizen in America should know a reasonable amount of mathematics and also that he should be trained to use it in an intelligent manner. To put it in another way, some of our people will play the role of producers of mathematics while others for the most part will be merely consumers.

[11] Reeve, W. D., "The Universality of Mathematics," *The Mathematics Teacher*, Vol. 22, pp. 71–83, February, 1930. See also Hotelling, Harold. "Some Little Known Uses of Mathematics," *The Mathematics Teacher*, Vol. 29, pp. 157–169, April, 1936, and Hogben, Lancelot. *Mathematics for the Million*. George Allen and Unwin, Ltd., London, 1936. Issued in America by W. W. Norton, Inc., 1937.

It should be our concern to see that these consumers are taught to appreciate the work of the producers just as some people who may not be able to produce music may nevertheless appreciate good music when they hear it.



If many of the present-day ills of this country could be traced to their origin, the cause would be found to be a lack of knowledge of the mathematics underlying the situation or a failure to appreciate its important implications. Much of our failure to understand many of the problems we have to meet in everyday life is due primarily to a lack of knowledge and appreciation of the simplest kinds of arithmetic. This explains why it has been so easy for unscrupulous salesmen to hoodwink so many housewives and also why it is so difficult to get people to investigate the ultimate cost of certain types of buying on the installment plan. It also explains why people have lost their life savings because of a lack of ability in or appreciation of some of the most obvious types of informational arithmetic.[12]

Francis P. Garvan puts the case very

[12] Judd, C. H., "Informational Versus Computational Mathematics," *The Mathematics Teacher*, Vol. 22, pp. 187–96, April, 1929.

well for arithmetic when he says

One of the ancients said: "*Remember, O Stranger, Arithmetic is the first of the sciences and the mother of safety.*" History has never been able to contradict that statement, and yet today, in America, in Catholic education and non-Catholic education, arithmetic is the most neglected of all sciences. Today a child's study of arithmetic stops at the age of 12, and many of our ills today can be attributed to the fact that we are trying to live men's lives and a great nation's life based upon a child's understanding of arithmetic. Let us in Catholic education carry the study of arithmetic right straight through the high school and through the college, through business arithmetic, bookkeeping, cost accounting, algebra and geometry, in so far as they supplement the understanding of the highest arithmetic, cost accounting, values and so forth, so that when a boy or a girl comes out of our high schools or our colleges he or she will be able to not only budget his income and his outgo, but to break down that budget and bring home to himself a realization, for example, of what cost prohibition, what cost foreign trade, what cost a credit system controlled abroad, what cost corruption in government—national, state and city—what cost tariffs, high or low, what cost crime and its punishment, what cost the administration of our charities, as well as what cost his home—how much of that cost is usury—how much of that cost is dishonesty of construction or dishonest materials—what cost neglect of our own health or the health of our children, and so forth and so on.[13]

The problem for the teacher is to equip himself as well as he can in all these fields so that he can bring out the proper correlation when the time comes. And it ought to be emphasized that the teacher of mathematics, other things being equal, is the best qualified to say when and where mathematics will be most helpful. However, any correlation of subjects in the schools should afford an opportunity for the right kind of cooperation between the various fields.

[13] Garvan, Francis P., *The Need of Catholic Leadership in Scientific Research*, speech before the Friends of the Catholic University of America at a dinner at the Knights of Columbus Club Hotel, Eighth Avenue and 51st Street, New York City, February 1, 1933.

There is nothing new in the idea of correlation. It is a practice which good teachers have always observed. However, it must not be done without care and thoughtful planning. Just as mathematics can make important contributions to physics, so physics can be used to clarify and enrich mathematics. In reality, if we are to give our pupils a well-rounded education, it is not a question of either mathematics or physics but a question of precedence and emphasis.

In this connection, Professor Moore says

At the present stage of development of both mathematics and science, the relationships between them and their mutual services are best illustrated in the fields previously mentioned, namely astronomy, physics, and physical chemistry. There exists no branch of science, however, in which some mathematical procedure is not found essential, Moreover, the natural evolution of all scientific theory is in the direction of increasing use of quantitative methods. It seems inevitable that the applications of mathematics in the more descriptive sciences should be enormously extended in the future. Many indications of the processes leading to such extension can be found in recent scientific advances in various fields. For example, the application of statistical methods in the biological and sociological fields is steadily increasing. In view of the fact that in the recent past no special mathematical preparation was regarded as important for workers in these fields, such an increase is noteworthy. We also find cropping up in these same sciences quite unexpected and rather startling instances of the possibilities of mathematical application. An example of this is found in the curve of healing of a wound developed by Carrel and du Noüy during the late war.[14]

It should be said here that before teachers can properly correlate mathematics with other fields, they ought to learn how to correlate the various parts of mathematics. They should first learn how and where arithmetic and informal geometry can be correlated, how and where algebra

[14] Moore, Charles N., "Mathematics and Science," *Science*, Vol. 81, January 11, 1935 p. 31.

may be best correlated with arithmetic and informational geometry, and so on. Unless we can do this, there is small chance that we can successfully correlate mathematics with science, music, the arts, and other applied fields. We should not make the mistake of breaking down subject matter lines before teachers are qualified to make the proper contacts and to explain the significant relationships between subjects to their pupils.

The trouble is not so much that mathematics has not been "integrated," as it is that it has not been well taught. Even without "integration" mathematics might have proved interesting and even useful to most pupils if their teachers had been properly trained for their work.

Whether we like it or not, an attempt is being made to correlate the various subjects in the curriculum. While this is at present confined largely to the elementary school, there is a strong tendency to do the same thing on the secondary level, and in colleges and universities as well. The mathematics department and the department of the four natural sciences at Bryn Mawr College have recently agreed to revise their general plan of undergraduate work in order to "break down the imaginary barriers between the sciences." Dr. Conant's recent proposal of "Professors at Large," professors "without portfolio," is obviously intended to protect Harvard University against the "disintegration" of the University into separate "compartments."

The idea at best is sound. Teachers of mathematics should direct and not block such a movement. We need to show how the study of mathematics will help one to be a better student of science, music, or the arts. We need also to teach mathematics as a method of thinking so that our pupils will go out into life able to think better because they have learned to think in studying their mathematics.

Some may say that the task of correlation is so great as to be impossible of realization. This need not be true if we begin on a small scale and try each year to educate ourselves better in each field.

In commenting on the suggestion made by Dr. Conant, Chancellor Chase of New York University recently said:

> The particular suggestion made by Dr. Conant gives formal status to what naturally is informally happening in many instances in American universities; whether the "roving portfolios" suggested by Dr. Conant are in themselves making a material contribution to its solution must be determined by experience. It may be doubted whether many of these professors will rove far from the home base, but any *intelligent* plan which tends to break down *undue departmental specialization*[15] should be welcomed and watched with interest.[16]

If what Dr. Chase has said is true for the university, it is all the more important for secondary education to break down the barriers that exist between subject matter fields. However, this uniting of related fields must not be done in any haphazard way. Too often incidental learning is accidental learning.

It is conceivable (and perhaps even desirable) that traditional organization of subject matter will be replaced by new learning units which have been prepared to meet the needs of the pupils. But if this is done we will still have to guard against mechanical correlation. However, in most of the integrated programs, as now set up, subject matter is not utilized until needed. When this happens such large blocks of subject matter have to be brought in that the teacher, because of the internal sequence of the content material, does the task mechanically for the pupils, and understanding and all sense of relationships between fields are lost. Thus, opportunistic teaching may have as great or even greater weaknesses than systematic development of subject matter.

Take the case of the social studies and mathematics, for illustration. If, in the history classes, the teachers were more

[15] The italics in both cases are mine.
[16] *New York Times*, February 10, 1935.

familiar with the great names in mathematics such as Newton and Descartes, they might spend less time on the war lords and war heroes and give some attention to men who have probably affected our lives more for good than has Napoleon. Men of science like Pasteur should be given consideration in proportion to their influence on mankind. Again, teachers of mathematics if they knew more about the social studies might teach graphs when they would serve the needs of the pupils in the geography or the civics class. A recent unpublished study by Hellmich[17] shows that in a certain area of the social studies textbooks for the seventh grade presuppose a knowledge of graphs for which the textbooks in mathematics have not prepared the pupils prior to the seventh grade. Who is to teach this material and where? Here is a chance for the social studies teachers and the mathematics teachers to cooperate. There still remains the question as to who is the best fitted to teach a given topic. Such questions should be answered experimentally and with great care. On the higher levels such a cooperative plan is already under way.[18]

Finally, we can probably make great progress in improving instruction in the secondary schools if we can only work together in a cooperative way. Much of the stagnant situation we find in the schools at the present time is owing to a lack of understanding between teachers and educators and can be removed by intelligent action.

The new monograph *Numbers and Numerals—A Story Book for Young and Old*[19] which has just been sent gratis to all subscribers of *The Mathematics Teacher* is a good sample of teaching material that can be used to advantage by both teachers of mathematics and the social studies alike.

[17] Hellmich, Eugene. *The Mathematics in Certain Elementary Social Studies in Secondary Schools and Colleges*. Bureau of Publications, Teachers College, Columbia University, New York, 1937, Contributions to Education No. 706.

[18] "Collegiate Mathematics Needed in the Social Sciences," Report prepared for the Social Science Research Council, *The American Mathematics Monthly*, Vol. 39, pp. 569–77, December, 1932.

[19] Smith, David Eugene and Ginsburg, Jekuthiel. *Numbers and Numerals—A Story Book for Young and Old*, The Bureau of Publications, Teachers College, Columbia University, New York, N. Y., 1937. Price 25¢ postpaid.

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Mathematics—A Language

MATHEMATICS, too, is a language, and as concerns its structure and content it is the most perfect language which exists, superior to an venacular; indeed since it is understood by every people, mathematics may be called the language of languages.—C. DILLMAN.

Individualized Instruction in Algebra

By DANIEL W. SNADER
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INTRODUCTION

IT HAS BEEN said, time after time, that the most neglected pupil in our high school is the brilliant boy or girl. There seems to be a leveling down of the classwork to meet the "tempo" of the average child and partially, if not wholly, neglect the superior students upon whom our future leadership will depend.

Good teachers of mathematics have always recognized the problem of individual differences but have done very little to effectively cope with the problem. To assign several more problems to the bright boy who needs less time to do the required amount of work expected of the rest of the class, is often the means of deterring him from working up to his capacity. Why, says he, must I do more for my A (mark) than the others who cannot work quite as fast? I believe that here again we have been guilty of a serious crime against the pupil of superior ability. Some educators declare that in our schools we are preparing pupils *for life*, while others are convinced that education is not a preparation for life, but *the very essence of life itself*. In view of either of these doctrines, how can we justify our dealing with the general case just cited? In the world about us this same pupil sees that man is rewarded according to his ability. He watches a modern business enterprise conducted on a "piece-work" basis. He sees the skilled mill laborer complete a piece of work and receive the reward (as pay) directly in proportion to the quality of his labor and the speed of performance. The more the worker does and the better he does it, the greater is his reward. If a man is a fast worker and finishes before his fellow employees, he is not told to do a few extra jobs (without extra remuneration) until the others "catch up," so to speak, but is immediately allowed to go on with the next job, thus being permitted to attain a wage goal proportionate to his own individual ability.

Let us examine, more closely, the basis for determining the pay for the "piece work" contract or job just mentioned. The highest price paid for a completed job is a function of the *quality* of the workmanship and the total pay is a function of both the *quality* of the workmanship and the *speed* of the performance.

To be more specific let us see how this relationship, $W = Q.S.P.$, affects our skilled laborers in our industrial world. (W) represents the pay John Doe earned during the past week. (Q) (which is never greater than one) is determined by the number of flaws or mistakes in the finished product. (S) represents the worker's speed and is determined by the number of specific jobs performed in unit time. (P) stands for the price in dollars paid for each job completed. Thus we see that a worker can receive the maximum price for a job if he meets the requirements of first quality workmanship and that his total wage depends upon his rate of doing work, as well as the quality of that work. Since (P) is the standard price per job, it remains constant throughout the consideration.

In the business world, this procedure is accepted as a fair and justifiable basis for determining the earning power of a skilled laborer. With seven years of experience as a skilled laborer and assistant foreman in a modern industrial plant, I know that each worker is rated according to the above described scheme. Each is given an opportunity to work up to his native capacity and his weekly wage is based upon what he does and how well he does it.

I am convinced that one of the great problems facing our schools today, and

consequently our nation of tomorrow, is that we have recognized the changes in quality and number of the pupils entering our high schools as compared with the situation of ten to fifteen years ago, but we have done very little of real value in coping with the situation. Mr. William Betz of Rochester, N. Y., speaking before the weekly conference of mathematics teachers at Teachers College last summer, indicated the gravity of this very situation. In a Democracy we believe in an educated proletariat. Equal rights and opportunities for education should be afforded the children of all the people of our land. Doubtless, the United States has been providing these opportunities. However, it is doubtful whether we can long agree to the assumption that "all men are created free and equal." Any school man and more especially any teacher knows that at least the last part of that statement is absolutely false. The differences in native ability and initiative as well as physical structure of the present-day students of our schools are apparent to any one who seeks the truth and has the opportunity of courageously trying to teach these pupils on the hypothetical basis that they are equal.

It has been the practice of our schools of recent years to "pass" every pupil from year to year—or in other words to graduate every student who has established an *attendance record of twelve consecutive years*. The advocates of this policy declare that every student has the right to pass and to taste of the glorious feeling of success. I do not wish to attack this theory as stated but to attach a very important clause to it. Every student shall be passed, promoted, and graduated, who has earned this right by meeting a certain minimum standard of accomplishment. This theory, which has long been the basis for reward in the business world, the world into which most of our youth are absorbed in their later struggle for a livelihood, seems to offer us a remedy for our present condition without the careless and dangerous practice of hurdling minimum standards of accomplishment and subsequent destruction of individual initiative.

But administrative authorities say, "how can we insist upon our high school students meeting modern minimum requirements before we graduate them, when we know that they differ so widely in initiative and fundamental preparation? Our schools are overcrowded and we cannot keep them in high school more than four years!"

This, indeed, is a serious administrative problem but it must be solved, *soon*, if we wish to avoid a much inferior generation of public school graduates. This practice of mass promotion, without any reference to educational standards, is bound to affect our professional world, too. Pupils graduated by our *State-aided schools*, are accepted into *State Colleges and Universities* where, with inadequate preparation, they enter the professional training courses—and who, after drawing all the political strings, are graduated into our professional ranks with a very meager sort of training.

To point out the weakness of our present system of mass education is not sufficient to bring about any reform. To merely recognize that pupils differ widely as individuals, does not meet this challenge. Wise and thoughtful experimentation is our only hope.

How shall we as mathematics teachers meet this problem of individual differences without creating insurmountable difficulties for the school administrators and without sacrificing the minimum standards of accomplishment?

Preliminary Considerations

About eight years ago, I began investigating methods of presenting algebra to students who were not classified either by intelligence ratings, reading ability, or previous academic records. The administrators of our school system were facing the same conditions that handicap other schools. The enrollment of the high school

had increased approximately forty per cent in the past eight years. The result had been that in spite of an increase in our high school teaching staff, the size of our mathematics classes had nearly doubled. The general economic conditions had prevented any further increases in public taxation for the support of schools. Consequently, the only recourse possible was to devise, through experimentation, new methods for coping with the situation.

Objectives of the Experiment

These are some of the problems we had to consider:

1. The large class enrollment.
2. The tendency toward lowered standards.
3. The differences in ability of our pupils.
4. Elimination of pupil failures.
5. The testing program and what to do with it.
6. Recognition of increased administrative problems.

The specific objectives of the experiment were:

1. To allow pupils to progress as fast as their individual abilities permitted.
2. To maintain satisfactory standards of accomplishment as the basis for passing the course.
3. To use "mastery of the subject matter" as the criterion for marking.
4. To provide a system by which pupils learn "how to study, gain confidence in their own ability, and work up to their individual capacities."
5. To provide a system for objectively testing the degree of mastery attained in every unit of the course.
6. To provide a system in which guidance and not telling, plays the leading role.

How the Objectives of the Experiment Were Realized

Recognizing the need of providing students with simplified explanations of the usual textbook treatment of the fundamentals of algebra, it was necessary to "bridge the gap" between the formal textbook treatise of algebra, with its usual strange and technical explanations, and the modern work book or exercise book which, for the most part, offer little or no explanations at all.

Working on the basis that the textbook explanations are too difficult for many boys and girls studying algebra (owing probably to their individual language difficulties), I have arranged study sheets in simplified language to accompany each unit of the course. This study-guide or explanatory material indicates:

1. The objective for the unit.
2. New terms and their meanings.
3. New principles and processes.
4. Explanations of the work, i.e., hints and suggestions, aids in mastering the unit, and particular errors to be avoided.
5. References to textbook explanations.
6. Supplementary practice material.

The following is a typical "study sheet" for one of the units of algebra.

Literal Equations

1. *The Object* in studying this unit is to learn to solve a literal equation for any one letter in terms of the other letters and numbers.

2. *New Terms and THEIR MEANING:* A literal equation is one in which some or all of the known quantities are represented by letters.

For example,

$$\frac{a-bx}{ax-b} = \frac{a}{b}.$$

3. *Procedure Used in Solving Literal Equations:* To solve literal equations, the same method is used as in solving numerical equations.

Order of Procedure

1. Clear the equation of fractions, i.e., multiply both members of the equation by the smallest number which contains all of the denominators as factors.
2. Perform any indicated multiplications.
3. Use the addition and subtraction axioms, i.e., transpose to one side of the equation all the terms which contain the letter for which you are to solve.

4. Factor both sides of the equation.
5. Divide each side of the equation by the coefficient of the letter for which you are solving.
6. Check your result, i.e., substitute the value you found for the unknown, in the first equation.

NOTE: Not all equations need every step in the procedure suggested. Omit any step that is not needed and take the next one in order.

Example:

$$\frac{x-a}{b} = \frac{x-b}{a}.$$

1. $a(x-a) = b(x-b)$,
2. $ax-a^2 = bx-b^2$,
3. $ax-bx = a^2-b^2$,
4. $x(a-b) = (a-b)(a+b)$,
5. $x = \dfrac{(a-b)(a+b)}{(a-b)} = (a+b)$.
6. $\dfrac{(a+b)-a}{b} = \dfrac{a+b-b}{a}$,

$$\frac{b}{b} = \frac{a}{a},$$

Check $1 = 1$.

4. Explanatory material will be found in most elementary algebras.

5. Supplementary practice material will be found in the card file in front of the room. Ask your teacher about it.

PRELIMINARY PREPARATION

At the beginning of the term each pupil is given a complete set of these explanatory outlines for the entire course. They are thus given a birds-eye-view of the material to cover. The teacher carefully explains to them *how* and *what* they are expected to accomplish. After studying this explanatory material and perhaps some of the outline, the pupils are ready to apply what they have learned.

SETS OF PRACTICE CARDS

On the regular size filing card (3″ by 5″), I have recorded many problems of the type to be mastered in each particular unit of the course.

Each card has two identifying numbers as in the following diagram:

Unit 9.	Card 1.
Solve the following equation for x	
$\dfrac{ax+b}{c} = \dfrac{cx+2a}{b}$.	

The number in the upper left corner refers to the particular unit of the course for which the cards are intended. The figure in the upper right corner is one which may be classified as the *call number* or the reference number used to identify the problem when checking the results. For each set of cards there is an answer card of a different color on which the answers for all problems of the group are recorded.

The answer card for unit no. 9 is arranged as follows:

Unit No. 9—Solution of Literal Equations	
1. $(2ac-b^2)/(ab-c^2)$	16. $(2b+3a)/(a+2)$
2. $-3b$	17. $(2b-3a)/(b-3)$
3. $2a/3$	18. $-a-b$
4. $(3a-5b)/3$	19. a/b
5. $(3c-2b)/2$	20. b/a
6. $(4-c)/2c$	21. $-c/a$
7. $2/3$	22. $-4b$
8. $18a$	23. m/b
9. $(9-24y)/2$	24. $7a$
10. $a/3$	25. $-a$
11. $2ab$	26. $4a-3c$
12. $(2a-b)/6b$	27. $3b$
13. $(abc)/(b+c)$	28. $-n/m$
14. $(a^2b)(a+b)$	29. $(c-4)/2$
15. 1	30. b/n

HOW THE CARD SYSTEM OPERATES

After one or more pupils think they understand the explanations of the problems on the "study sheet" for unit no. 9, they ask for practice cards dealing with "Literal Equations." They are given a set of cards as described above.

This individual or small group of students pass to the rear of the room and start solving these problems, while the

slower pupils are still studying the explanation on the "study sheet" and in the textbooks to which they are referred. When they have solved five or six problems—and feel as though they thoroughly understand the solution of this type of equation, they are given a mastery test on the unit. (A description of these tests will follow.)

By this time the rest of the class is ready for the problem material. Each is given a different card. They are at liberty to solve the problems on paper at their desks or to make use of the blackboards for this purpose. As they solve their problems, they rise at their seats, remain standing until recognized, and then state their card number and the corresponding answer to the problem. The pupil rises—when recognized, he says:

Unit 9. Card No. 1.
Answer $x=(2ac-b^2)/(ab-c^2)$

The teacher has the answer card and checks the results as they are read. If the answer is correct, he is given another card; if incorrect, he is seated again and checks over the solution to find the error. If the answer is incorrect the second time, he then passes to the blackboard where the teacher can point out (at a glance) the error in the solution. Now, as with the superior group, when these pupils have solved correctly at least five of the problems on the cards, and feel able to solve any problem of this kind, they are allowed to take the mastery test on the unit.

The Testing Program

Purpose and Description of the Materials

Briefly, the tests are designed to meet the objectives of a modern algebra course. In addition, they have several other outstanding advantages over the usual type of tests.

1. They are designed to permit the teacher to use the same test sheet for re-examination purposes in cases where certain pupils do not meet the required standards of mastery on the first trial.[1]
2. They are easily and objectively checked.
3. They fit in exceptionally well with the basic theme of the procedure "*Individual Progress According to Pupil Ability and Initiative.*"

It will be easier to explain these advantages by first examining at least two representative types of tests—one dealing with the manipulative phases of algebra and the second dealing with problem analysis of various applications. These two tests are used as examples not because they are more important than the rest but because they represent the results of a random selection from the regular classroom files.

Unit Test No. 9. Solution of Literal Equations

Name:.....

Test Card No..... Score.....

In the following literal equations, solve for the letters indicated.

1. $\frac{b-x}{c}=3-\frac{c-x}{b}$ Solve for ( ) Ans.....
2. $A=1/2h(b_1+b_2)$ Solve for ( ) Ans.....
3. $V=1/3Bh$ Solve for ( ) Ans.....
4. $i=prt$ Solve for ( ) Ans.....
5. $T=2\pi R(H+R)$ Solve for ( ) Ans.....
6. $S=1/2gt^2$ Solve for ( ) Ans.....
7. $I=\frac{E}{R+r}$ Solve for ( ) Ans.....
8. $V_1P_1=V_2P_2$ Solve for ( ) Ans.....
9. $\frac{1}{f}=\frac{1}{f_1}-\frac{1}{f_2}$ Solve for ( ) Ans.....
10. $\frac{E}{e}=\frac{R+r}{r}$ Solve for ( ) Ans.....

Unit Test No. 25. Motion Problems

Formula, $D=rt$

Name.....Test Card No... Score....

1. Two boy-scouts left from their camp, one going east and the other going west. The scout going east traveled ( ) times as fast as the other. In 6 hours they were ( ) miles apart. How fast did each scout travel?

Ans.....E. Ans.....W.

[1] Six years of experimentation, has shown me that 70% mastery is a satisfactory standard for passing a unit.

2. John and George are two aviators ( ) miles apart. How long would it take them to meet if the former travels at the rate of 90 miles per hour and the other at ( ) miles per hour?

Ans.....hr.

3. ( ) hours after a freight train, going 40 miles per hour, left a certain station, it was followed by a passenger train at ( ) miles an hour. How long will it take the passenger train to overtake the freight train?

Ans.....hr.

4. A bus and an automobile start at the same time, at the same place, and travel in opposite directions. The bus travels at the rate of 40 miles an hour and the automobile at the rate of ( ) miles an hour. In how many hours will they be ( ) miles apart?

Ans.....hr.

5. Two cars located six hundred miles apart, travel toward each other. One car goes at the rate of 50 miles per hour, while the other one goes at the rate of ( ) miles per hour. In how many hours will they be ( ) miles apart?

Ans.....hr.

6. Mary and Jane start from the same place and travel in opposite directions. Mary's rate of travel is ( ) times Jane's rate of travel. In ( ) hours they will be 420 miles apart. How fast did each travel?

Mary's rate=.....mi. per hr.
Jane's rate=.....mi. per hr.

7. A car, running ( ) miles per hour, left Athens at 9 A.M. toward Cleveland. ( ) hours later, a second car traveling at 50 miles per hour, followed after the first car. What time will it be when they meet?

Ans.....

8. ( ) hours after a messenger traveling at the rate of 8 miles per hour, had left headquarters the General decided to cancel his message. How fast did a second messenger have to travel in order to overtake him in ( ) hours?

Ans.....mi. per hr.

9. (A) traveling ( ) times as fast as B, leaves the same place at 10 A.M. traveling due North. (B) leaves the place at ( ) P.M. and travels at the rate of 25 miles an hour due South. In how many hours will they be 300 miles apart?

Ans.....hr.

10. A young couple want to take a Sunday trip in the country. At an average rate of 30 miles an hour, how far may they go, so they can return in ( ) hours?

Ans.....mi.

These two unit tests are typical of the entire course. Each is constructed to test the important specific objectives of the unit.

The parentheses in each problem of unit No. 25 may be filled with any numbers which are suitable and may be so varied that no two pupils need to be given the same ones when they take the test. This means, also, that if a pupil fails to achieve a minimum standard of mastery of the unit, the teacher may give individual attention to the pupils errors, supply remedial work,[2] and then by simply changing the numbers to be inserted in the parentheses of this same test sheet, reexamine the pupil's mastery of the unit.

While the pupils know what types of problems they will be expected to solve, nevertheless by this unique device of varying the specific details, no two tests will be identical.

There is no hope for copying; the only way out is to acquire a knowledge of the fundamental concepts, specific techniques, and manipulative phases peculiar to the unit.

However, to insert numbers at random in the parentheses of each test, would be a simple matter. But, this very simplicity becomes drudgery when the tests have to be examined. In order to overcome this difficulty, I have resorted to the card system. When a pupil is prepared for a certain test he is given a *Test Card* upon which the specific numbers to be inserted in the parentheses of each problem of the test are recorded. For every *Test Card* there is a corresponding *Answer Card*, upon which the proper results of the test are recorded.

Operation of the Testing Program

When a pupil has completed[3] a certain unit of work, say No. 25, he is allowed to take the test.[4] He is given a *Test Card* like the following:

[2] The card system previously described.

[3] Completed—in the sense of adequate preliminary preparation as described under this heading.

[4] Thirty to forty minutes is the usual time required to finish a test. However, a full period is allowed.

Unit No. 25	Test card: A
1. 3, 160	*6.* 2, 5
2. 900, 120	*7.* 25, 2
3. 2, 60	*8.* 2, 6
4. 55, 650	*9.* 3, 1
5. 40, 90	*10.* 10

After copying these numbers in their places on the test sheet and recording the test card number *A* in the space provided at the top of the test, the pupil returns the card to the teacher and proceeds to solve the problems on yellow examination paper provided for this purpose. As he solves the problems he places the result on his test sheet after the word *answer*. When finished with the test or at the close of the period, he turns in both the test sheet and the yellow paper upon which he solved the problems.

This requirement of turning in all solutions of the problems, serves as a guarantee against copying and against the establishment of poor habits and incorrect methods of solution.

Sometime before the next class period the teacher takes the Answer Card No. *A* and checks the results by a mere comparison of the answer on the card with those on the test sheet. By glancing through the solutions on the yellow paper, compensating errors, and incorrect methods may occasionally be detected. The *Score* for the test is placed in the space provided at the top of the sheet.

The papers are returned to the pupils at the beginning of the next class period. All corrections must be made before going on to the next unit of work. If a pupil's score is such as to fall below an arbitrary standard of mastery, then he needs to reach that standard by further study and reexamination before attempting the succeeding unit. This procedure eliminates the danger of future retardation so often caused by inadequate basic preparatory work.

The score on any unit is the average of all the scores a pupil makes on it. This provision serves to check pupils against hurrying through a unit and taking a test with the hope of getting by—so to speak. They are not slow in learning that this procedure very often penalizes them and is not at all conducive to a high rating. They realize that in order to make a good grade they must be careful not to take a test before they are actually ready for it. The responsibility for success rests squarely upon the shoulders of the pupils themselves. I have found them very willing to carry this responsibility, too. This training is transferable and a good way to teach it, is to give exercise to it, in as many school and life situations as possible.

The pupils are at liberty to take their test sheets home with them and consult them in connection with the study sheets of the various units of the course. While they know what type of problems are going to be in their tests, yet the test cards previously described will so vary the required responses that copying and minor deceits are practically impossible, and mastery of the fundamentals of the units can be objectively measured.

If we set up a worthwhile list of objectives for a course, why not let the pupils know what we think is important? Why keep this test material, especially the unique type just discussed, away from the pupils as though it were sacred and not to be seen before we "*spring it on them*" at the end of a unit? I believe that if a problem or question on a test is worth knowing, then the pupils should be allowed to study it. When pupils learn the meanings, purposes, and implications of the tests we give them—then the veil of fear usually surrounding the term (due to the mysterious atmosphere so often enshrouding their preparation and administration) will be pierced and pupils will begin to study mathematics with the competitive spirit of a game from which the winner emerges as a result of his own individual ability, initiative, and constructive effort.

Provision for Class Discussion

To provide an opportunity for general class discussion of concepts, terms, and definitions—and to prevent any anti-social tendencies of individual members of the class, I have found it pleasant and profitable to set aside one period of the week during which no tests may be taken but during which general class discussions of topics covered, with special reference to their general and mathematical meaning and applicability, are given a clearer and more meaningful interpretation.

Many interesting side lights on the historical, appreciational, and recreational phases of mathematics can thus be woven into the course. Pupils desire this sort of informal discussion on the interesting *Sideshows* of the subject and a wise teacher will find it an opportune way of promoting a genuine pupil-love-affair with the subject of mathematics.

Summary

Please observe that in the procedure I have discussed:

1. *Guidance of pupil activity* and not *teacher telling* plays the leading role.
2. Pupils learn how to study, gain confidence in their own abilities, and work up to their individual capacities.
3. The problem of individual differences in ability among pupils of mathematics is handled in a practical manner; practical to both the pupil and teacher.
4. The pupils are allowed to progress as fast as their individual abilities permit. The "tempo" of progress is regulated by the temperament of the individual.
5. The degree of mastery on any one unit or on the entire course is arrived at objectively.
6. "Mastery of the subject" is the criterion for marking.
7. Satisfactory standards of accomplishment must be met in order to pass the course.
8. Pupils are interested in the work—because of the fact that real satisfaction follows success.
9. The stigma of failure is removed—all pass *when* and only *when they meet a certain minimum standard of accomplishment.*
10. The reward, as in the industrial world, is proportional to the amount and quality of the accomplishment.
11. Its flexibility makes it adaptable to large or small classes at a minimum of administrative difficulty.

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Recent Changes in Point of View Relating to the Teaching of Arithmetic*

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In many school subjects tradition is being routed by scientific studies in education. The teaching of arithmetic is at present a most active field in a most active period of education. In 1924, Buswell and Judd summarized the investigations relating to the teaching of arithmetic that had been reported at that time. Since then, more than four times as many experiments have been made as were available at that time. Moreover, the rate of the appearance of investigations is still increasing. The findings of these studies have caused us, who work in this field, to reorganize our philosophy and our classroom procedure to be in conformity with the evidence that is now available. This reorganization has taken various forms. In the main, the reorganization in arithmetic has occurred along three lines, namely: the broadening of the aims, the selection of learning exercises because of their social use, and the arrangement of these exercises in the order of ease for learning. I want to examine with you those recent changes in point of view and in classroom practices that I judge most important.

First, I want to make a plea for a larger amount of quantitative thinking on the part of pupils than usually results from the way elementary school arithmetic is now taught. We have long sought to acquaint children with the facts and the processes of arithmetic as though these were ends in themselves. They are not that, but, on the contrary, they are but means to other and relatively more important ends.

While the needed facts and procedures must be better taught than is now usual, this result will not be realized unless opportunities for the children to do quantitative thinking about things that are real and vital to them are provided. In other words, the facts of arithmetic should be related to quantitative situations continuously while they are being learned so that the children will be conscious of their need of these facts and so that they will understand how each result is found. I think that enough of these facts should arise from concrete situations to show that concrete situations give rise to all of them.

For example, 7 and 6 are 13 not because the combination looks wrong with any other sum, but because whenever 7 and 6 are counted together the result is found to be 13 regardless of the unit in which they are each expressed. Furthermore, the concepts 7 and 6 must be clear before 13 has any meaning. The movement should be back and forth between the abstract facts and the concrete situations that use these facts. Too much of our teaching is upon the low level of specificity and not enough upon the generalizations that give meaning to these specific elements.

Our present insistence upon isolated drill on each fact has been due largely to two factors. The first is that standardized tests overemphasize computation. These tests have come to play too large a role in determining curricular items. Too many teachers devote too much of their time to items similar to those of the tests rather than to situations that use needed elements. The second cause of insistence upon specific learning is due to the erroneous assumption of the bond psychology that there is little transfer of training from one fact to others, even though these facts are all highly related;

* A paper read at the summer meeting of the National Council of Teachers of Mathematics at the N. E. A. Convention in Portland, Oregon, June 27, 1936.

and hence that each must be explicitly learned by deadening repetition, if it is important enough to be known. Hence the drill theory for teaching arithmetic has prevailed, notwithstanding the fact that survey tests show that successive groups of children of a given age level are not becoming more accurate in the fundamentals as the years go by. For example, the recent results of the Courtis tests show that 8th grade children have less than 80% accuracy in the fundamentals. We can get a high degree of accuracy if we teach for it and test soon enough after drilling but we are now beginning to realize that permanency of learning is more closely related to uses now and subsequently than to the number of repetitions during learning. Furthermore, the chances of transfer to other situations, that have not been specifically prepared for, are far greater if the facts and skills of arithmetic are taught and retaught with the meanings that result from using these facts in concrete situations. But even if the teaching of the facts with meaning did not facilitate their learning, I should vote for their being so taught for the sake of the contribution to the habit of thinking quantitatively that their learning in this way makes. After all is said, quantitative thinking is the most important aim of our teaching. This is at present especially true since number is daily occupying a more central position in our economic and political life. Decisions in both these fields are made on the basis of the understanding of number. For example, the vote of each one of us next November will be determined largely by our interpretation of the quantitative phases of the issues.

Arithmetic is a coherent, unified science rather than a multiplicity of details. The thread of unity that runs through the whole science should be recognized at every step. This is best done by relating the facts of arithmetic to experiences. Thinking quantitatively results from a fund of experiences and not from any number of unrelated details. The spirit of any science is far more important than its minutiae. Of course, a learner must wade through enough of the details to arrive at the spirit. However, the teaching should be centered upon uniting these details into a science.

Since arithmetic is such a science of related facts and procedures, an activities program in and of itself is not sufficient to assure learning. Such a program is most valuable in supplying concepts and clarifying them. It must be supplemented by summaries that assure that the work on any grade level is thoroughly enough done. Understanding alone, though valuable, is not sufficient. The children should be induced to make that understanding outstanding.

As I see it, the major purposes of the teaching of arithmetic are these, though the order is not significant:

1. To develop skill in computation and manipulation with accuracy and reasonable speed.
2. To give power to solve problems using these skills.
3. To develop the ability to interpret accurately quantitative expression thought, heard or read.
4. To further the ability to understand the quantitative phases of social problems, e.g., the ability to see the fallacy in fallacious slogans.
5. To give economic information that will help children later in their own economic problems.
6. To accomplish the above aims in such a way as to give joy to the learner.

Now, while none of these purposes can be isolated and developed by itself without reference to others, they each need explicit attention. For example, the computational aim serves the purpose of the informational one by gradually unfolding the meaning of number and the number system. It serves the further purpose of making clearer the concepts of numbers and especially of relatively large numbers. For example, one knows 72 better by knowing it is 8 9's or 6 12's, than is gained by the serial meaning alone. On the other hand, informational arithmetic is helpful

to computational arithmetic in that it helps to clarify the processes.

Do you know that to some of my colleagues the computational phases of arithmetic are all there is to it. If that were all, arithmetic could well be called a tool subject. Too long have the purposes of arithmetic been subordinated to the tool-phases. Experimentation has been concerned chiefly with computational arithmetic and not with the more important phases of the subject. Likewise, surveys reveal the fact that in most schools 85% to 90% of all time devoted to arithmetic is concerned with increasing proficiency in computational arithmetic. I take the percentages above from a survey made rather widely two years ago in the schools of the State of Minnesota.

Some of the titles of articles referred to above are: The Analysis of the Processes; The Identification of Errors and Their Correction; The Maintenance of Skills; To Check or Not To Check; and the like. Seldom is insight into the uses of arithmetic touched. Lately we are coming to see that the former are but the tools of arithmetic and that arithmetic has for its true purpose values far beyond these skills —values to which these skills must contribute.

"The arithmetic," says Dr. Buckingham, "that has for its purpose the furthering of these skills alone is woefully one-sided. The teacher that sets out to do that and that only defeats her own purpose." These skills will not long remain in any mind unless reasons for them and exercises in using them are also a part of the program and not a small part at that. In my own classes in our school about the only fact of measure that is remembered is that there are 5280 feet in a mile. The numbers 231, 2150.42, 30¼, 320, 160, and the like, have no meaning to most of the students though these students have been at one time thoroughly familiar with each one of them. The child that learns the facts and manipulations of arithmetic in a mechanical way is not learning arithmetic in the sense that he has an understanding of quantity when such an understanding is needed. Furthermore, such a child will not be so good a computer as one who has learned the facts and the processes of arithmetic with meaning. In other words, a program that is formal and sets out to secure mastery of the facts of arithmetic by sheer deadening drill not only offers a meager course, but it fails to accomplish the mastery at which it aims. I am voting against such a program because it is so meaningless and so unproductive.

Furthermore, psychologists are in rather general agreement that unless the child has formulated his own goal and sees himself advancing towards this goal, learning is at best a very expensive affair. Hence, mastery of the facts of arithmetic must be accepted by the child as his personal project, if he is to secure this mastery at all.

So much for the change in objective relative to teaching arithmetic. I want to look next at the change in the types of learning exercises children are asked to do.

The time was not very far back when this topic was not deemed important. Now it is of major importance. The children have a right to have the tasks that they are asked to do filled with information that is valuable to them now and that will be increasingly valuable later in their work-a-day sphere and at the same time give them a more precise basis for their intellectual thinking about the nature of the world in which they live and of the society among whom they shall live. Accordingly today the teacher of arithmetic should scrutinize carefully every learning exercise to assure herself that the doing of the exercise will be of the most advantage to the children both now and later in helping them to make the adjustments that they will need to make. In other words, the exercises should be selected with the definite purpose of giving children insight into the uses of the facts and the processes of arithmetic and at the

same time, as full an understanding of the mathematical principles underlying these facts and processes as the children can have at that time.

There is grave doubt that the miscellaneous lists of problems with each one about a separate situation are of the most value. Problems in real life seldom come to one written in a book. Related problems about a single situation promise greater returns. The arithmetic for a whole grade could well grow out of a few situations such as renting and furnishing an apartment; the buying, housing and maturing of a flock of 500 pullets; the equipping of a store; the planning of an automobile trip to the sea-shore; and the like. Any one of these would take the class far afield and cover a wide area of arithmetic. The learning of arithmetic would then have more purpose than results from isolated problems alone. The conclusion of the investigation by Harap and Mapes to determine the efficacy of an activities program for teaching arithmetic is that "under conditions of good teaching an activities program based on real situations in school and social life that incorporate the basic steps of a grade may be taken with considerable assurance that these basic steps will be mastered." However, such a program must recognize two truths, namely, that there are a very large number of facts, processes and manipulations that will be needed by the child. It must furthere recognize that there is an order of ease in learning the mechanical phases of arithmetic. Accordingly, any activities program should be directed and guided into those activities that recognize these truths. Moreover, summaries must show that the basic steps of the grade are being mastered.

The tendency of late both of expositions in the field of the teaching of arithmetic and in the textbooks for children is to have the work much more simple than was the case even a few years ago. The educational philosophy behind this change is that the children should understand first and learn later. Indeed there is extreme doubt whether learning in advance of understanding is at all permanent. In fact, all evidence points to the truth that learning is far more rapid and much more dependable if understanding accompanies the learning. Consequently, even in the learning of the addition facts enough concrete illustrations of the meaning of these facts should precede the learning of the facts. This program will compel teachers to continuously teach and reteach these facts based upon situations that require them.

Arithmetic as formerly taught was too difficult for most children. Yet, arithmetic is not a difficult subject. The trouble has been that the subject matter has not been adapted to the ability of the children. The growth has been forced and not natural. There has not been enough effort to bridge the gap between counting things, measuring, and the like, and abstract numbers. Consequently the facts and processes were learned by meaningless drill. This drill part of the program coming too early, as it did, was the cause of most failures. On the other hand, there should be a rich field of concrete number experiences between counting and manipulation of numbers. The natural growth into quantitative thinking should be developed before exercise for mastery. If one responds correctly to quantitative thinking, it is due to the possession of a rich fund of experiences numberwise and not due to ability to make automatic responses. Arithmetic has suffered from the lack of rich content of meanings because the tool phases have predominated. What is needed is more varied experience numberwise. Tests show that the real cause of inability to apply arithmetic in real problems is a lack of understanding of the situations giving rise to the problems.

Another tendency, of which I want to speak this afternoon, is to change the grade placement of the topics. Many topics are being placed in later grades than has been usual. Among these are long-division,

operations upon fractions, and percentage. An examination of the test results of the children in our Campus Elementary Training School shows that the number of errors of the children in a topic is least from three to four years after that topic was first taught. If this fact is taken into account as the topics are advanced into grades later than is usual, it means that each major topic must also be continued as much beyond the initial treatment as its initial treatment has been delayed. This means that some of the topics formerly thought of as belonging in the elementary grades would not be introduced until the junior high school, and that the maintenance program must be extended beyond the junior high school grades.

There would be no loss if this were done. There is little purpose in attempting to teach a topic for which there is an inadequate background of preparation and for which there are few needs when later on there will be ample preparation and more natural uses on the level of the children's experiences. The evidence shows that it is at this point that children's failures are due. The problem work that calls for a technique is not furnished to the children. As a result, failures and discouragements are numerous. Accordingly, the children that learn the technique lack the ability to use it in subsequent work. Children's growth numberwise should be natural from the first grade on. Then there would be much fewer failures with the mental blocking that results. There are enough number needs on each grade level to assure this natural growth. While I am convinced that work in arithmetic should grow out of appropriate activities wherein the need for number is present, I do not favor leaving out this stream of our heritage from any grade. The investigation of Dr. Nila Smith shows that first grade children have many real number needs. Likewise, the investigation of Buckingham and McLatchy shows that first grade children are able to begin number work. Hence neither from lack of need of number nor from unreadiness to learn number is there any reason for delaying number work to later grades.

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The Young Pascal

By Harold Maile Bacon
Stanford University, California

Versatility is not inevitably the companion of genius. It is not altogether common to find a man who is at the same time a clever experimental physicist, a creative mathematician, an inventor with an eye to money-making, a gifted writer whose artistry places him among the foremost French stylists, and a religious philosopher of singular originality and ardor. Blaise Pascal was such a man. He could write an important treatise on the vacuum as well as produce those incomparable examples of controversial literature, the famous *Provincial Letters*. He invented the first adding machine of practical consequence and tried (in vain) to realize a profit from its sale. In the fragments of his projected apology for the Christian Religion, left unfinished by his death at the early age of thirty-nine, are to be found many evidences of his mathematical genius as well as of a remarkable piety and zeal. A clear and complete picture of his early life and education would not only be of rare interest, but it could not fail to contain many suggestions of value to the modern teacher. It is indeed a pity that such incomplete information is available. The story is soon told, but it is well worth the telling, and perhaps it holds some inspiration or lesson for our own times.

Blaise Pascal belonged to a family of provincial officials. Although it could boast a title of nobility granted by Louis XI in recognition of the faithful services of one Etienne Pascal, an important fiscal officer in the King's entourage, it was of parlementary rather than noble condition, and the Province of Auvergne provided places in its local magistracy and revenue offices for many of the younger members of the family. The Pascals were well-to-do, substantial people of considerable local importance, and had for generations audited the accounts of the province or presided over its courts.

Another Etienne Pascal was President of the *Cour des Aides*, a court having jurisdiction in all matters pertaining to indirect taxation, when his son, Blaise, was born at Clermont-Ferrand on June 27th, 1623. This strong-minded and talented magistrate had married in 1618 Antoinette Begon, the pious daughter of a family of the merchant class. A son born in 1619 lived only long enough to be baptized, but 1620 saw the arrival of a daughter, Gilberte, who was to become the biographer of her famous younger brother and sister, Jacqueline, born in 1625.

This was a period of scientific discovery and advance. The first half of the seventeenth century was to see the death of Francis Bacon, the work of Galileo, Kepler, Descartes, Roberval, and Fermat, the birth of Newton, Huygens, and Leibniz. We shall find Etienne Pascal and his son taking their parts in the work of the age. But in spite of the spirit of true scientific thought, there was still a wide-spread faith in astrology, "spells," witchcraft, signs, and omens. Even Kepler had to make astrological predictions to help to earn his living, and it was only by exerting considerable influence that he was able to save his mother from conviction of witchcraft. A curious story is told of the infant Blaise Pascal by his niece, Marguerite Perier,[1] which illustrates how even a man of his father's sagacity was forced to take account of the prevailing superstitions.

According to this story, when Blaise was between one and two years of age, he fell into a strange sort of languor attended

[1] Marguerite Perier, "Mémoire de la vie de M. Pascal, écrit par mademoiselle Perier, sa nièce" in *Oeuvres de Pascal* ed. by Brunschvicg and Boutroux, Paris, 1908, 14 vols., vol. I, pp. 125–136.

by two surprising symptoms. The sight of water threw him into convulsions, and, although he enjoyed the caresses of his mother or of his father separately, their approach together sent him into transports of childish rage. This illness lasted for over a year during which time he grew so much worse that his parents began to despair of his life. Everbody said that this malady was the result of a "spell" cast over the child by a woman who had received charity from his mother. Although his father was skeptical, he seems to have sent for the woman and to have wrung from her a confession of the sorcery. She was able to explain how the spell could be removed, but it would require the sacrifice of the life of another to whom the enchantment could be transferred. Fortunately an animal would do. The anxious father offered his horse. But, the witch remarked, the thing could be accomplished at much less sacrifice: a cat would suffice. So a cat was produced. When thrown out of a window only six feet from the ground, it instantly died. Next morning a poultice was made from three leaves each of three different herbs gathered before sunrise by a girl less than seven years of age. Upon application of this poultice to his body the child fell into a coma from which he awoke—just at midnight—completely cured.

Unlikely as this incident appears, it may well symbolize the spirit of the times, while the illness itself was the forerunner of that ill health which persisted throughout so much of Pascal's life.

When Blaise was but five years old his mother died. The father, a man of about forty, thenceforth lavished his care and affection upon his three children. Two years later, in 1631, he sold his magistracy at Clermont, and moved with his family to Paris where he planned to devote his time to the education of his children, especially his son. He would permit no one to help or to interfere with his original plan of teaching the boy. Although the Jesuit schools were then flourishing and providing what was regarded as a well balanced and methodical course of instruction, Etienne Pascal would have none of it. He preferred to keep Blaise at home and to teach him in his own way. He failed to count upon his own insufficiency in many branches of knowledge. He knew the Law as a former official, could use Latin about as well as most educated men of the day, was acquainted with mathematics and physics, and was a fervent believer in experimental science. He made up for the gaps in his knowledge and for his lack of experience by a method of teaching which was of his own personal invention. His basic principle was that the child's lesson should always be entirely within his grasp, easily and completely understood. A glimpse of the whole subject under consideration was to suggest simple and easily discerned general laws and to arouse curiosity. From these general principles were to be deduced explanations of particular facts observed or questioned by the pupil. Etienne Pascal would make no attempt to force or strain an inquiring mind.

That the youthful Blaise had an inquiring mind—not easily restrained by his father's notion that the child should be "held well beyond his work"—is evident from the biography written by his sister Gilberte.[2] He wanted to know the reasons for everything, and if good ones were not given he would seek them for himself and not give up until he had a satisfactory explanation. For instance, we are told that when he was eleven years old he noticed that a china dish struck by a knife produced a loud sound which ceased when the hand touched the dish. He was not content until, after several experiments, he had discovered the reason. He then wrote, no doubt with parental encouragement, a little essay on sound.

[2] Gilberte Perier, "La vie de Monsieur Paschal, escrite par Madame Perier, sa soeur, femme de Monsieur Perier, conseiller de la Cour des Aides de Clermont" in *Oeuvres de Pascal*, vol. I, pp. 50–114. This biography, written soon after his death, is the chief source of information concerning his early life.

There was an incessant inquiry into the origin and nature of things in general, into their properties and their uses. For example, what is grammar? How does it happen that all languages are communicable from one country to another? What of the extraordinary effects of nature such as gunpowder exploding in a cannon? It was expected that these inquiries would last until Blaise was twelve years old when he was to be put to the study of Latin and Greek. He was also to acquaint himself with Spanish and Italian, and always by methods and according to rules imposed by his father. When he should have mastered languages, he was to take up mathematics. According to this plan he would have been about fifteen or sixteen years old when introduced to geometry, and should by that time have reached a maturity and orderliness of mind properly fitting him to appreciate this absorbing subject. The whole scheme was intended to accustom the child to seek out knowledge for himself while guided in broad paths of learning, to render account of his work to himself, and never to take preconceived notions and hypotheses for truths.

While this method of education suited the boy's nature admirably, while it developed his good qualities and nourished his originality, it overlooked many important matters. History was neglected. Every idea was a personal discovery of the pupil and appeared to him as his own exclusive property. Such a lack of historical perspective encourages a man to exaggerate the consequences of his own ideas and to disdain the work of others to whom he feels he owes nothing. Who can say that Pascal did not at times exhibit evidences of a lofty egotism? "He who reads the *Thoughts* of Pascal should give a thought to the president of the *Cour des Aides* of Clermont-Ferrand who taught his son so well while teaching him so badly."[3]

That the method had surprising effects upon the lad is clear if we can believe an often repeated story[4] told by his sister Gilberte. One day his father happened to enter the play room unobserved. What was his surprise to find the boy of twelve engaged in drawing figures upon the stone floor with a piece of charcoal. Although he had not been allowed to study Euclid, when questioned the boy explained that he was trying to prove that the sum of the angles of a triangle is two right angles. His father then drew from him step by step the method by which he had used his "bars" (lines) and "rounds" (circles) to arrive at this thirty-second proposition of Euclid. Overcome with emotion, Etienne Pascal withdrew, and soon Blaise was given mathematical books to study. He had, no doubt, overheard in the conversation of his father's friends some references to geometry, and, not content to regard lessons in geometry as simply a promised reward for proficiency in Latin and Greek, had set out to investigate the subject for himself. This story may be largely the product of an adoring sister's imagination, and yet there must be at least some element of truth in it. She was older than her brother and was taking the place of a mother and a housewife. A young woman matured beyond her years by this experience, while perhaps possessing strong family partiality, must have seen clearly the remarkable ability of the boy. Whether in later years she dramatized this ability in some such imaginary incident, or whether the story is strictly accurate in all details, unlikely as this may seem, cannot be definitely said. However, there can be no doubt that young Blaise showed such proficiency in mathematics that his father, perhaps surprised and gratified at this result of his educational method, felt constrained to revise his plan to withhold mathematical studies until languages had been mastered.

Etienne Pascal had found his level in Parisian society. He was admitted to a

[3] Fortunat Strowski, *Pascal et son Temps, deuxième partie, l'histoire de Pascal*, Paris, 1907, p. 9.

[4] It appears on pp. 53–56 in vol. I of *Oeuvres de Pascal*.

circle of scientific men such as Roberval, Carcavi, Le Pailleur, Desargues, and gifted amateurs of high spirit. The central figure of the little group was Father Mersenne, a clever and agreeable Minimist friar who was in close touch with Galileo, Descartes, Torricelli and other scientific men of the day. The evident genius of young Blaise prompted his father to introduce him to this circle which was the immediate ancestor of the French Academy of Sciences. Here, although he was but in his early teens, he took part in work and discussions. And here, too, he found a school of manners. The right to speak depended upon the amount of information. Older men who knew little of a subject had to be content to listen to younger men who knew much. Such experiences led him to attempt to judge everyone by purely intellectual standards. It did not take him many years to discover that the world does not employ such a standard of judgment, for he observes in the *Thoughts*, "Rank is a great advantage, for it gives to a man of eighteen years of age a degree of acceptance and respect which another man can scarcely obtain by merit at fifty. Here is a gain, then, of thirty years without difficulty."[5]

It must not be supposed that this gifted youth led a life of solemn study with no amusement. While his father was a former magistrate and a *savant*, he was not solely a meditative philosopher. He was an intimate friend of the versatile and volatile Le Pailleur who ran about France and England, changed his religion from Protestant to Catholic and back again, taught himself mathematics, sang songs—he is said to have sung eighty-eight songs during one gay evening—danced, and perpetrated practical jokes upon unsuspecting friends. The mere fact that a man of this character was an intimate friend of the family is enough to dispel the picture of an austere cultivation of the intellectual at the expense of all other powers. A rare cordiality and familiarity characterized the family life. President Pascal was a most affectionate father. Gilberte was a sagacious and practical young housekeeper who, when away from her household duties, took a quiet place. Jacqueline, though much younger, was much more noticed.

The Pascals had visited Auvergne only once, in 1636, since their establishment in Paris. Most of their interests were to be found in the capital where their fortune had been invested in municipal bonds. But unfortunately Richelieu's policy of opposition to the House of Hapsburg had involved a considerable drain on French finances, and in 1638 in order to meet certain exigencies, the interest rate on the Parisian bonds was arbitrarily cut. This of course embarrassed and angered Etienne Pascal who associated himself with a group of protesting investors. Unhappily there was some show of violence, and two of the leaders were arrested and summarily removed to the Bastille. Pascal was able to escape and to take refuge in Auvergne. He risked return only when Jacqueline was dangerously ill with the smallpox. In the meantime the pathetic situation of the three children had struck the fancy of the fashionable world. In February, 1639, Jacqueline was encouraged to take part in the play, *L'Amour Tyrannique*, which was performed before the Cardinal. She so charmed Richelieu that his attention could be favorably attracted to the gifted son and exiled father, and Jacqueline was instructed to tell her father to return.

The evident ability and well known integrity of President Pascal so appealed to the Cardinal that he resolved to send the former magistrate to Rouen to act as Intendant. Like most people, the French did not like to pay taxes. In 1639 the national treasury was seriously depleted, and in spite of the most determined efforts of the Minister, the provincial parlements failed to squeeze any more money out of the people. The local officials sympathized with their neighbors and did nothing. The

[5] Blaise Pascal, *Oeuvres de Pascal*, vol. XIII, pp. 240–241.

Cardinal, resolving to put an end to such nonsense, created the office of Intendant whose occupant was to act as a sort of vice-regent in the matter of tax collections. His power was as great as his popularity was minute. Normandy was one of the most unruly of the provinces and needed a firm hand to bring it up to the mark, and the capable and courageous Etienne Pascal was the man for the job. Accordingly he arranged to remove his family to Rouen in the autumn of 1639.

Meanwhile Blaise had been thinking about mathematics and joining in the discussions of Mersenne's circle of friends. He had undoubtedly studied carefully many of the mathematical books in his father's library, and had also made himself familiar with the new geometrical methods of Desargues. His father's educational method had encouraged original work on his part, and at the age of sixteen he was to be found engaged in the production of a new theory of the conic sections, a forerunner of which appeared in 1640 as *Essay pour les coniques.* In this little handbill was announced a lemma which has become famous as "Pascal's Theorem" and amounts to the statement that the intersections of the three pairs of opposite sides of a hexagon inscribed in a conic are collinear. The youthful investigator acknowledged his indebtedness to the methods of Desargues who had endeavored to reduce the properties of the conic sections to a small number of propositions. The improvement over the old method of considering each conic as a separate curve was in treating these curves simply as various perspectives of a circle. That Pascal had carried this new method much farther than had its inventor was at once acknowledged by Desargues who gave the "mystic hexagram" the name *La Pascale.* Great praise immediately fell to the lot of the sixteen year old mathematician. Everyone was enthusiastic—everyone but Descartes who grumbled to Mersenne, "It is just as I thought! I had not read half the little essay on Conics by M. Pascal's son before I saw that he had taken most of his ideas from M. des Argues, and this was confirmed soon afterwards by his own confession."[6] But there can be no doubt of Pascal's originality. The *Essay pour les coniques* was the announcement of a treatise which he was preparing. It concludes with the modest statement

> We have several other problems and theorems and several consequences deducible from the preceding, but the mistrust which I have of my slight experience and capacity does not permit me to advance more until my present effort has passed the examination of able men who may oblige me by looking at it. Afterwards, if they think it has sufficient merit to be continued, we shall endeavor to push our studies as far as God will give the power to conduct them.[7]

Mersenne reported that in this treatise Pascal had deduced the properties of the conic sections from his "mystic hexagram" in four hundred corollaries. While the work was never published, Leibniz knew of it and gave a summary of it in his letter of August 30th, 1676 to Etienne Perier, Pascal's nephew.

This discovery of Pascal indicates clearly his view of mathematics. It rests upon the idea that the properties of a complex figure may be considered as a modification of those of a more simple figure. The effort of his father to have him always take a comprehensive and unifying view of everything helped to save him from the all too common error of the student who so concentrates upon some small details that he misses the simplicity and unity of his subject.

The removal to Rouen when Blaise was seventeen soon put a stop to these extended researches in pure theory. There his father plunged into the business of straightening out the chaotic finances of

[6] Rene Descartes, Letter of April 1, 1640, to Mersenne in *Oeuvres de Descartes* ed. Charles Adam and Paul Tannery, Paris, 1899, 12 vols., vol. III, p. 47.

[7] Blaise Pascal, *Oeuvres de Pascal,* vol. I, pp. 259–260.

Normandy, and his son was expected to help him as much as possible. But his native genius, combined with the encouragement which his education had given to original work of all sorts, soon (1643) led to the idea of an "arithmetical machine." Before this adding machine could be finally perfected, more than fifty models had to be constructed, and it was not until some years later that a patent was secured which forbade the copying or construction of the machine by anyone else.

The achievements of Pascal in the fields of mathematics, physics, literature, and philosophy, and the story of his later life are too well known to be repeated here. But it is well to pause to consider the remarkable effect which the manner of his education must have had upon him. It fostered original investigation—even of simple things well known to the world of his day—above all else. We see its flowering, if not in the apocryphal "discovery" of geometry, certainly in the theorem of the hexagon, at an early age. Undoubtedly his genius would have asserted itself under any circumstances, but it was surely awakened and nourished by that schooling so well adapted to his needs. We should be fortunate indeed if today we could make available such an opportunity for every talented young man and woman.

The Circle

Few things are perfect: we bear Eden's scar:
Yet faulty man was godlike in design
That day when first with stick and length of twine
He drew me on the sand. Then what could mar
His joy in that obedient mystic line—
Then calculating with a zeal divine
He called π 3 point 14159
And knew my lovely circuit $2\pi R$.

A circle is a happy thing to be—
Think how the joyful perpendicular
Erected at the kiss of tangency
Must meet my central point, my avatar—
And perfect as I am, yet only three
Points are needed to Determine Me!

—Q. E. D.

—Christopher Morley at the Annual Luncheon of the Mathematics Chairmen at the Astor Hotel in New York City on February 27, 1937.

National Council Members Attention!

The National Council of Teachers of Mathematics will hold its summer meeting at Detroit, Michigan, on June 28, 29, and 30 in connection with the N. E. A. President Hildebrandt is preparing an interesting program which will appear in the May issue of *The Mathematics Teacher*.

Greetings to My Friends of the National Council*

By Herbert Ellsworth Slaught
University of Chicago, Chicago, Illinois

When I was asked to send this message to you, I was warned that it would not sound natural without a joke and a laugh from me. Well, here is a joke that has never been in print, not even in the *Literary Digest*. Way back in the early eighties the President of Colgate University, who was an austere man of powerful physique and commanding presence, and who considered himself the personal guardian of the college boys, was the originator of many a unique saying current on the campus, of which the following is a sample:

Our freshmen had cut classes all day in order to attend a forbidden maple sugar festival at a neighboring farm-house, and the President had called us into his study where the boys were seated about the room, mostly on the floor. As he was discussing the principle of such infractions, my classmate Harding, a mild-mannered young fellow, arose and attempted to explain and justify our behavior. Whereupon the President rose partly to his feet and, leaning over his desk, with one arm outstretched towards Harding, fairly shouted in stentorian tones—"Sit down, young man, sit down! Some day you may be president of this University, but not now, not now!" Ha! Ha! Ha! The boys did not laugh at the time, but they did for many a day thereafter.

Here is another taken from real life at the University of Chicago. After the World's Fair of 1893, the Midway, remained unlighted and unimproved for a long time, and it was considered dangerous to cross after dark. One of the head professors who was attending an evening meeting on the other side, was warned by his wife to beware of hold-up men. He replied by brandishing his club-like cane and daring anyone to attack him.

When half-way across the Midway, a stranger brushed against him. The professor instantly felt his watch-pocket and found it empty. Whirling about with cane upraised he demanded: "Here, give me that watch!" whereupon the stranger delivered up the watch.

That evening as the professor was boasting to his wife of his exploit, she calmly remarked: "Why, Thomas, your watch is on your dresser in the bedroom." Ha! Ha! Conundrum: How would you advise the professor to get the watch back to its owner?

Now having enjoyed our laugh, and with no fear on my part of being "held up" or ordered by you to "sit down," I must say a few things in a serious vein as to the mathematical situation in America: Twenty years ago the *American Mathematical Society* was the only nation-wide organization in America dealing solely with mathematics, and its concern was entirely with

* This is a translation of the record that was made and given by the Allied Radio Corporation at the annual dinner of the *National Council of Teachers of Mathematics* in the Grand Ballroom of the Palmer House at Chicago on February 20 where because Dr. Slaught was not able to be present his birthday was honored by speeches and the record. The Allied Radio Corporation said:

"Heretofore, it has been impossible to make a professional recording outside of a recording studio. The equipment used to make such transcriptions was extremely large, cumbersome and very delicate; in fact, so delicate that most recording studios had their recording table set in concrete so that vibration was at a minimum. Recording on aluminum has been the only way of making transcriptions outside of a studio.

"The address of Dr. H. E. Slaught was recorded on a regular professional acetate disc. This is the first time that we know of a recording of this type being made on portable equipment in a home. The equipment used to make this transcription is entirely portable and requires about twenty minutes to set up for operation. It is fairly simple in operation and does not need a recording or sound engineer to operate it."

the field of research. In 1916, the *Mathematical Association of America* was organized with the chief purpose of serving the interests of collegiate mathematics. In 1920 the *National Council of Teachers of Mathematics* was organized with the chief purpose of serving the interests of mathematics in the elementary and secondary schools.

These three organizations, together with the valuable contribution being made by *School Science and Mathematics* in cooperation with the other basic sciences, now cover the whole range of mathematical interests in America. Each of these groups is incorporated and rests upon a sure and strong foundation.

The membership in the Society and the Association is approximately what one might think it should be in proportion to the clientele of each, but this is not the case with the National Council, whose clientele should easily warrant a membership twice as large as it now possesses.

The officers of the Council are fully aware of their responsibility in this matter, and already measures have been taken and are now under way, looking to a great enlargement of the Council membership. I should now like to propose the renewal of the slogan which some of us adopted just before the depression overtook us, namely, "Our goal—10,000 members by 1940."

This is not a fantastic proposal, but it will require long and strenuous application to accomplish it, or even to approach it. Meanwhile the surest way to win the interest of non-members is to magnify the usefulness of *The Mathematics Teacher* and of the *Yearbooks*. Every teacher of mathematics in the elementary and secondary schools should find the National Council a bulwark of strength in his profession.

The National Council occupies a unique position in the field of mathematical pedagogy and every member has the opportunity to contribute toward the betterment of teaching in whatever phase of elementary or secondary mathematics he may be most interested.

For example, the teaching of arithmetic is a fertile field in which there is ample room for improvement, and the topics of senior high school and junior college are always open for expert pedagogical attention.

The Mathematics Teacher affords a medium for the promulgation of ideas in the realm of elementary and secondary mathematics that no teacher in this realm can afford to be without. Moreover, such possession is a mark of professional standing.

The *Yearbooks* sponsored by the Council bring to the teacher the very latest and best considerations in the line of our mathematical interests. There are now eleven such volumes.

If all these things are good for our four or five thousand members, they are also good for twice that number. So here's to our slogan: "Ten thousand members by 1940."

Now, having had our serious chat, may I close with a personal word of thanks to all of you who have from time to time sent me such warm greetings, and with a word of special appreciation to those who have made possible this present recognition of my interest in the welfare of the National Council.

Notice to Subscribers!

Look out for the expiration date of your subscription. It is printed on the outside wrapper of each number of *The Mathematics Teacher* which you receive.

THE ART OF TEACHING

A NEW DEPARTMENT

Optional Topics in Plane Geometry

By CLARA LARSON

Schurz High School, Chicago

WITH THE change in the philosophy of education and with mathematics on the defensive, I found it necessary to change my method of teaching geometry and to stimulate interest in the subject. I found it necessary to connect geometry with daily life and to have activities suited to the needs, interests, and abilities of the pupils.

Early in the course I decided to make some provision for individual differences by setting up a list of optional projects. I received my first inspiration for this from "Provisions for Individual Differences, Marking and Promotion"—a government bulletin (1932) No. 17, U. S. Department of Interior, Office of Education. Page 364 gives a list of "Practical Pupil Opportunities." I believe my "Bridge and Steel Construction" project is original. While working on the congruence of triangles, to show the rigidity of the triangle, I asked a pupil to bring to class a triangle made of thin pieces of wood. Another pupil conceived the idea of making such a triangle with pieces from an old erector set. He also brought in a quadrilateral which made from an erector set easily changed its shape, also a quadrilateral of the same size with the addition of a diagonal rod. This of course was perfectly rigid. This boy then mounted these on a large cardboard and added a picture of the traditional bicycle showing the use of the triangle in its construction. Before he was through he had a nice collection of pictures of bridges and steel construction work. He brought to class a magazine given to employees by the United States Steel Corporation. This had a nice little article on triangles and trusses as used in steel work.

While we were studying angles another pupil became interested in the transit. This boy looked up the history and use of the transit and made a simple transit. He also made a study and reported on other field instruments such as the sextant and vernier.

A girl who found formal geometry very difficult but who was exceptionally good at art work said that when she drew a face she always blocked in the angle of the nose and shape of the head with straight lines. She worked out a neat little project on "Geometry in Art." She made one set of drawings of faces and heads showing the construction lines and another set showing the finished product. She also worked on the construction of pentagons, the five-cornered star, hexagon, and the six-cornered star. This, no doubt, has been done before, but the point is that the girl herself had the idea of connecting her art work with geometry.

We have another bulletin board for "Geometry in Advertizing"—geometrical trademarks, copies or original. Construction lines were left on the first copy. Such trade marks as Kraft's cheese, Illinois Central Railroad and the Good Housekeeping Bureau seal of approval make good examples. This offers a big field and I had contributions from the whole class. The art work on them was good.

One of my students is now working on the relation of mathematics to music, another on the relation of mechanical draw-

ing to mathematics. He found that the draughtsman's ways of drawing two lines parallel or two lines perpendicular with the celluloid triangle were based on geometrical theorems. While we were on the "Sum of the angles of a triangle" another pupil became very interested in Einstein's theory and he worked up a respectable report on Einstein. It developed that another pupil in the same class had met Mr. Einstein when he was visiting in Chicago.

I think that my classes this year like geometry better because of these contacts with the rest of the world and certainly it is more interesting for me. After it is once started this interest gains momentum. Of course, we have our theorems with their formal proofs, but we don't pound at them every minute.

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EDITORIALS

Importance of Belonging to the National Council

A YOUNG man who is ambitious to secure a better position than he now holds as a teacher of mathematics walked into the office of *The Mathematics Teacher* recently to inquire if any good positions were available. During the conversation he revealed that his membership in the National Council of Teachers of Mathematics had lapsed and when offered a subscription blank said that he could not afford the two dollars necessary to subscribe for the magazine. He did, however, manage to dig up a quarter to pay for a copy of "Numbers and Numerals" which is monograph number one of the new series on "The Contributions of Mathematics to Civilization." There is no doubt that the case of this young man is typical. Many teachers of mathematics who wonder why they do not get better positions fail to realize that it is probably their apathy toward the very organizations that might help them. Not all of them are so financially embarrassed that they cannot belong. They lack interest, enthusiasm and group loyalty. Teachers who keep up with what is going on by reading the official journal and the *Yearbooks* and by attending as many meetings of their group as is possible are, other things being equal, most likely to be leaders or at least to be successful in their field. It is to be hoped that members in addition to the regularly appointed State Representatives of the Council will do a little missionary work not only in their immediate circle, but also at any sectional meetings which they attend.

Are You Interested in Your Officers?

AT THE last meeting of the National Council of Teachers of Mathematics in Chicago only a few more than two hundred out of over forty-five hundred members of that organization cast votes for their choice of candidates. Such indifference to the welfare of the Council is shocking. If our members are not more interested in the Council than that, it is conceivable that the control of the organization can be obtained by a mere handful of votes. Just suppose that the people in this country were proportionally as apathetic about casting their votes in a national election as we have been. Such an attitude might spell the death of our democracy. If there is anything that the Council can do to improve this situation, members who have ideas should communicate with the president, Miss Martha Hildebrandt. When such indifference becomes known it gives people who already think teachers are queer folk a chance to continue their accusations with some chance that people, who do not know the truth, will believe them.

What Group Consciousness Can Accomplish

AT A recent meeting of the *Oklahoma Council of Teachers of Mathematics*, a vote was taken as to their first, second, and third choices of textbooks for use in the state. It seems likely that the State Board will abide by this decision of the mathematics teachers or at least give very serious consideration to it. This is only one of many important problems that teachers have to solve that might be handled better if teachers only had a little more interest and enthusiasm for their own group. Just think what we might accomplish as a National organization if all mathematics

teachers joined the National Council of Teachers of Mathematics and then took an active part in its meetings, *The Mathematics Teacher*, the *Yearbooks*, and in all of our attempts to improve mathematical education. Right now, for example, we ought to cease our traditional efforts of talking about what ought to be done to improve the teaching of various subjects and get down to the main business of providing some worth while materials of instruction for our pupils. Many of us have been preaching the gospel of a general mathematics course beginning in the seventh grade and running through the senior high school and yet today there is not a single series of textbooks of this kind. If everyone who has any real interest in the cause of mathematics would join the Council and work for it, we could develop a rich source book of materials that would revolutionize the teaching of mathematics in the schools. There is no better time than now to act.

Poor Child!

College Professors:

Such rawness in a pupil is a shame.
Lack of preparation in the high school is to blame.

High School Teacher:

Good heavens, what crudity; the boy's a fool!
The fault, of course, is in the grammar school.

Grammar School Teacher:

From such stupidity may I be spared;
They send them up to me so unprepared.

Primary Teacher:

Kindergarten blockhead! And they call
That preparation. Worse than none at all.

Kindergarten Teacher:

Such lack of training never did I see;
What kind of woman must the mother be!

The Mother:

Poor helpless child—he's not to blame;
His father's folks are just the same.

—*The Western Pennsylvanian.*

IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

1. Adams, L. T. *History of mathematics.* Sierra Educational News. February 1937, p. 23.

A convincing appeal for the importance of the history of mathematics to the teacher of mathematics. A brief bibliography is included.

2. Allbritten, Pauline. *Construction problems and original proofs versus memorized proofs in geometry.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 3. February 1937, pp. 5–6.

Interesting comments on the teaching of geometry.

3. Barta, Dorothy. *A project for mathematics classes.* High Points. Vol. 19, no. 1. January 1937, pp. 56–57.

The author describes a project the purpose of which was to have pupils answer for themselves the question, "Where is mathematics used in daily life?"

4. Bracken, John L. *How science and mathematics are taught in the Missouri schools.* School Science and Mathematics. 37: 209–15. February 1937.

An address, delivered before the Annual Convention of the Central Association of Science and Mathematics Teachers, dealing with the number of schools in Missouri that give courses in the various sciences and in mathematics.

5. Campbell, Winona. *The teaching of arithmetic.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 3. February 1937, pp. 8–10.

A review of the articles by Dr. William A. Brownell and by Dr. Leo J. Brueckner, that appeared in the Tenth Year Book published by the National Council of Teachers of Mathematics.

6. Hartung, Maurice L. *Evaluating appreciation of the cultural values of mathematics.* School Science and Mathematics. 37: 168–81. February 1937.

A clear analysis of an extremely subtle problem. The treatment is given under four headings:

1. Some problems which mathematics teachers must face.
2. A tentative statement of some objectives.
3. Important assumptions.
4. Sample test items.

In summary the author states that "on the basis of these meager data we are not justified in drawing any general conclusions. We do have some evidence, however, that the ability to do logical reasoning as defined above is not being achieved very well by our present materials and methods of instruction. We have some definite evidence that a few pupils do reason logically and do know some of the principles of logical reasoning. We now face some very important questions concerning how we can teach so that *more* students will exhibit the behaviors involved in logical reasoning. . . ."

7. Lindberg, Viola G. *Mathematics and art.* Bulletin of the Kansas Association of Mathematics Teachers. Vol. 11, no. 3. February 1937, pp. 10–11.

A very enthusiastic report of a course that the writer took in the summer of 1936, at Teachers College, Columbia University, under Dr. Georg Wolff of Düsseldorf, Germany, on "The Correlation of Mathematics with Art, Science and Music."

8. Lorey, Wilhelm. *On Dieffenbach's method for the solution of biquadratics.* National Mathematics Magazine. 11: 217–20. February 1937.

An exposition of a memoir published in 1821, containing a method for the solution of biquadratics which is not well known in mathematical circles. The differences between the method under consideration and other methods are pointed out.

9. Quine, W. J. *New foundations for mathematical logic.* The American Mathematical Monthly. 44: 70–80. February 1937.

After a clear analysis of the main accomplishments and shortcomings of Whitehead and Russel's Principia Mathematica, the author suggests a method of avoiding the well-known contradictions, without accepting the theory of types or the disagreeable consequences which it entails.

10. Richeson, A. W. *Notes on an eighteenth century English mathematical manuscript.* National Mathematics Magazine. 11: 221–30. February 1937.

The writer quotes generously from the manuscript and makes interesting comments on it. The conclusion is that the "manuscript was probably prepared by one of the instructors at the Royal Naval Academy, while it was located at Portsmouth for use in his classes in mathematics. The text covers the elementary subjects of arithmetic, geometry, trigonometry, and applications to many allied topics."

11. Rickey, F. A. *On the projection of an angle upon a plane.* National Mathematics Magazine. 11: 209–12. February 1937.

Although mathematical intuition is an indispensable tool for the mathematician, yet there are instances in which intuition alone may be misleading. The author supports the above thesis by pointing out that "Most of us have the feeling that the projection of an angle upon a given plane is not equal to the given angle unless the plane is parallel to that of the given angle. Nevertheless it is true that any angle can be equal to its own projection upon a plane, provided only that the plane of the angle is not perpendicular to the given plane." The author proceeds "to determine the magnitude of the projection of an angle under given conditions, to determine its maximum and minimum values and the 'position' the given angle must assume in its own plane in order that it shall be equal to its projection."

12. Salkind, Charles. *A geometry class examines the Constitution.* High Points. Vol. 19, no. 1. January 1937, pp. 44–47.

An interesting illustration of the possible application of habits of analysis acquired in the study of mathematics to other fields of thought.

13. Schorling, Raleigh. *Needed research on the problem of the slow-learning pupil.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 2. December 1936, pp. 6–11.

A continuation of the discussion on the same topic in the issue for October 1936. This article contains a tentative list of facts concerning the dull and also a list of procedures that are appropriate to the above-mentioned list of facts. A sample unit of subject matter is also included.

14. Scott, Leota. *A review course in Parsons High School.* Bulletin of the Kansas Association of Mathematics Teachers. Vol. 11, no. 3. February 1937, pp. 6–8.

The author describes the topics taught in a review course in mathematics that is required of all the seniors in the high school of Parsons, Kansas.

15. Seidlin, Joseph. *Review of "A report on present tendencies in the development of mathematical teaching in Japan, by M. Kuniyeda."* National Mathematics Magazine. 11: 231–40. February 1937.

The reviewer's comments are so dextrously intertwined with quotations from the report that one gets a complete picture of the state of mathematical teaching in Japan. The Japanese writers themselves modestly proclaim "that despite the fact that Japan seemed to have been left some time in the past about 20 years behind the European and American nations in starting the movement for reforming mathematical teaching, she has made steady progress in this direction since 1918 until at last at the present time Japan may take pride in being devoted to assiduous duties on mathematical teaching, keeping her position on the foremost front of the mathematical education in the world and yet without being affected by the reactionary thought prevailing in various parts of the world."

16. Struyk, Adrian. *Geometrical representations of the terms of certain series and their sums.* School Science and Mathematics. 37: 202–208. February 1937.

By means of nine drawings, the author shows that "Certain elementary theorems on the summation of simple arithmetical series can be illustrated in a manner analogous to that used for the algebraic identity,

$$a^2+2ab+b^2=(a+b)^2$$

The illustrations show how geometric figures which represent the terms of one member of an equation can be assembled to form a geometric figure which represents the other member."

The following are a few of the arithmetical series the sums of which are represented by illustrations:

(I) $1+2+3+\cdots\cdots+N=\frac{1}{2}N(N+1)$

(II) $1+3+5+\cdots\cdots+2N-1=N^2$

(III) $1+2+\cdots\cdots+N+\cdots+2+1=N^2$

17. Tucker, Charles B. *The value of mathematics.* Bulletin of the Kansas Association of Teachers of Mathematics. Vol. 11, no. 3. February 1937, pp. 1–4.

A readable summary of the various reasons that are given when one is pressed for an answer to the question, "What is the value of mathematics?" Perhaps the best answer to that question is the quotation from the report of the National Committee on Mathematical Requirements, given in the article under review.

NEWS NOTES

The eighteenth annual meeting of *The National Council of Teachers of Mathematics* which was held at the Palmer House in Chicago on February 19 and 20 was by far the biggest meeting that the Council has ever held. There were over one thousand people present at the meeting and the Chicago mathematics teachers showed their loyalty by running their own attendance up to 650.

Two new features of the meeting were the three round table conferences on Saturday morning which were filled to overflowing and the discussion luncheon where people had to be turned away because there was no room left at the tables. A concentrated report of the things discussed at the luncheon will appear in a later issue of *The Mathematics Teacher.*

The following letter from D. K. Howell of Pawnee, Illinois, is encouraging to the editorial committee of *The Mathematics Teacher:*

Dear Dr. Reeve:

For a number of months I have wanted to send you my congratulations and thanks for the work you are doing in publishing such a splendid periodical as *The Mathematics Teacher.* You and your co-workers are rendering the field of mathematics, as well as us teachers, a valuable service. The many worthy features of this publication make it a magazine which every mathematics teacher, who desires to be on the "firing-line," should read and study regularly.

This comes from a desire to inform you that many of us subscribe for other reasons than to obtain credits on our certificates.

Respectfully,
(Signed) D. K. Howell.

The "Report of the Proceedings of the Second Mathematics Conference" organized by the Headmistresses' Association and the Secondary Education Board, has been published and copies of the report may be obtained from Miss Marion Goodale of the Baldwin School at Bryn Mawr, Pennsylvania, for one dollar.

The fourth meeting of the Men's Mathematics Club of Chicago was held at the Central Y.M.C.A. on January 15. Mr. Pope, a past president of the Club gave a review of current magazine articles relating to mathematics. Professor Logsdon of the University of Chicago talked on the topic "What Should Mathematics do for College Freshmen?"

The Women's Mathematics Club of Chicago was responsible for a most excellent exhibit of classroom work in mathematics as well as of other very interesting books and charts at the recent meeting of the National Council of Teachers of Mathematics in Chicago. Miss Ida D. Fogelson was the person in charge.

Heath Has a New President

Important changes in the management of D. C. Heath and Company, well-known educational publishers took place on February 18. The resignation of Winfield S. Smyth as president was accepted, and he was succeeded by Dudley R. Cowles, vice-president since 1934 and manager of the Atlanta office of the Company since 1909. M. B. Perry, who continues to hold the office of treasurer, succeeds Mr. Cowles as vice-president. At the same time the Board of Directors was increased to seven by the election of A. B. Wright, manager of the Chicago office, Carl McGannon, manager of the Dallas office, and J. S. Smyth, Manufacturing Superintendent for the Company. Other directors are Frank W. Scott, secretary, and Allen G. Odell, manager of the New York office.

Mr. Smyth, who was elected Honorary Chairman of the Board of Directors, has been connected with the Heath Company more than forty years, during which time he has been successively agent, assistant manager of the Chicago office, treasurer, vice-president, and, since 1927, president. He has been a director since 1910. Born at Casenovia, New York, in 1872, he did his college work at Stanford, and has been active in the affairs of the Heath Company ever since leaving college.

Dudley R. Cowles, the new president, has been a member of the Heath organization thirty years. Born near Williamsburg, Virginia, he graduated from William and Mary in 1895 and was head of the schools of Hampton, Virginia, until 1900, meanwhile teaching in the Virginia Summer School of Methods, and serving as president of the Virginia State Teachers' Association from 1898 to 1900.

After seven years with Silver, Burdett and Company, Mr. Cowles joined the Heath forces in Boston thirty years ago this month. Two years later, after the death of D. C. Heath, founder, an office was opened in Atlanta and Mr. Cowles was placed in charge. He has been there ever since. Elected to the Associate Board in 1920, he became a director in 1923, secretary in 1927, and vice-president in 1934.

Mr. Cowles has been active in southern educational and cultural affairs. He served as president of the Atlanta organization of the Drama League of America and on the National Board of Directors. For several years he was president of the Atlanta Writers' Club, and is now president of the Georgia Association of Phi Beta Kappa. Mr. C. D. Daniel, who succeeds Mr. Cowles as manager of the Atlanta office, has been with Heath for more than twenty-five years, during many of which he has been assistant to Mr. Cowles as manager of the Southern territory, with headquarters in Birmingham, Alabama. A graduate of Vanderbilt, he taught in several southern institutes and colleges before engaging in the text book business.

Along with the several other changes in the Heath organization is the return to the editorial staff of Dr. José Padín, who for the past six years has been Commissioner of Education of Puerto Rico. Dr. Padín was for several years in charge of the Latin-American business of D. C. Heath and Company until he was borrowed in 1930 by Theodore Roosevelt, then Governor of Puerto Rico, to be Commissioner of Education. He served also at times as Acting Governor and as President of the University. His services to education have been recognized by an LL.D. from Haverford; Litt.D., University of Puerto Rico; and Ph.D., Dartmouth. In the Heath editorial department his attention will center in modern language and Latin-American publications.

Range Mathematics and Science Clubs

A joint meeting of the Range Mathematics and Science Clubs was held in Aurora, Minnesota at the High School on Thursday, February 4. This was a dinner meeting attended by eighty mathematics and science teachers, and several superintendents of schools.

The following program was arranged:

Welcome—M. W. Van Putten, Principal of Aurora High School

Responses—H. G. Tiedeman, President Range Mathematics Club; H. K. Savre, President Range Science Club

Violin Solos—Miss Helen Oresko accompanied by O. R. Olsen

Science Meeting, Cafeteria, H. K. Savre presiding.

- Plant Experiments—Ralph Backstrom, Aurora
- High School Museums—Supt. C. E. Hagie, Aurora
- Tour—Arrowhead Museums—lead by Dr. C. E. Hagie
- Round Table Discussions—Science Teachers

Mathematics Meeting, Library, H. G. Tiedeman presiding.

- Mathematics and Science of Geodesy—Lt. J. P. Lushene, Wash., D. C., U. S. Coast & Geodetic Survey
- Round Table Discussions—Mathematics Teachers
- Committee Report—Dr. A. D. Gillette, Superintendent of Schools, Eveleth, Minnesota

Dr. Gillette gave a brief summary of the plans of the committee formed to examine the purposes and recommend changes in the secondary school mathematics program. This committee is headed by Mr. W. B. Gundlach, president, Mathematics Teachers of Minnesota an Dr. John G. Rockwell, Commissioner of Education. Dr. A. D. Gillette, Superintendent of Schools, Eveleth, is also a member of this committee.

A Mathematics Exhibit consisting of a Mathematics Library, Instruments for Direct Measurement, Instruments for Indirect Measurement, a Mathematics Scrapbook, Portraits of Eminent Mathematicians, Models for Solid Geometry, and Calculating Machines from the Abacus to the modern Adding Machine was prepared for the visiting teachers.

Miss Josephine Sharp, instructor of mathematics in the Aurora senior high school and Mr. M. W. Van Putten, principal of the senior high school, were in charge of planning the programs and the necessary provisions for the dinner.

During the short business meeting of the mathematics club the members of the organization decided to hold the next meeting in Gilbert. Mr. Ole Schey, instructor of mathematics in the Gilbert High School will be in charge of program arrangements for the mathematics club and Mr. Edward Hribar, instructor of science will make the necessary arrangements for the science club.

The Mathematics Club also voted to send Mr. H. G. Tiedeman of Mountain Iron, president of the club since its organization three years ago, to the meeting of the National Council of Teachers of Mathematics. This meeting was held in Chicago on the 19th and 20th of February.

Curriculum Journal Becomes a Printed Publication

The *Curriculum Journal*, official organ of the Society for Curriculum Study, which began seven years ago as a mimeographed bulletin, became a printed publication with the issuance of the January, 1937, number, the first in Volume 8. Besides articles and other usual departments of an educational journal, the *Curriculum Journal* prints abstracts of curriculum research and

listings of recent courses of study. Its department of news notes is a thorough coverage of important curriculum projects throughout the United States. The *Journal* is printed in two columns and is in keeping with the present trend toward small magazines. The January number includes articles by David Snedden, Goodwin Watson, C. W. Knudsen, Edgar M. Draper, and A. V. Overn. The *Journal* is edited by Henry Harap who is the executive secretary of the Society for Curriculum Study. The publication office is located at the Bureau of Educational Research, Ohio State University, Columbus, Ohio. The subscription price is $2.50 a year.

Mathematics Seminar Offered at U. C.

The Mathematics Section of the California Teachers' Association, Bay Section, sponsored a seminar for teachers of mathematics, on the University of California campus in January. Mathematics 260 deals with applications of elementary mathematics and a discussion of subject matter selected from sciences related to mathematics.

The mathematics department staff of the University of California, headed by Mr. G. C. Evans, were instructors for the seminar. Meetings were held Wednesdays, beginning January 20, 4:15 to 5:45 P.M., in room 320 Wheeler Hall. Visitors were permitted to attend at any time.

"Figuratively Speaking" a new mathematics magazine was introduced to the students of the University High School in Oakland, Calif., in January, according to Geoffrey Beresford, editor. The price will be 5 cents.

There are 6 other editors. They are Marion Schlichtman, art editor; Lily Yamada, literature; Peter Gester, recreation; Howard Lea, personals; Doris McCrey, problems; and Tom Putnam, Business Manager.

The magazine will consist of brain twisters, fallacies, catch problems, optical illusions, believe it or nots, jokes, and several different kinds of puzzles.

The name of the magazine was suggested by Bruce Good in Miss Irene Lorimer's period 7 Trigonometry class.

Oklahoma Council of Teachers of Mathematics

The annual meeting of the Oklahoma Council of Teachers of Mathematics was held in the Central High School at Tulsa on February 5, 1937. John A. Venable presided.

A total of 152 members was reported for the past year. The Council started the new year with an enrollment of one hundred members. One hundred cards have been sent to other teachers of mathematics over the state urging them to join the Oklahoma Council and The National Council.

At the last meeting of the Council, committees were appointed to make a study of texts suitable for state adoption. At the 1937 meeting, the committees reported with recommendations. Additional recommendations were made from the floor. A written vote was taken for first, second, and third choice from each group of texts. The results of the voting will be sent to the state text-book commission. Also, a petition was signed and sent to the text-book commission urging consideration of the recommendations.

From contacts previously made with members of the commission, it was generally understood that they would attempt to select the texts for adoption from the Council's recommendations. The Council was very enthusiastic over the progress made along this line.

Seven members for 1937 were enrolled for the National Council of Teachers of Mathematics at this last meeting.

The following officers were elected for the ensuing year: John A. Venable, Tulsa, president; Marian Van Griethuysen, Elk City, vice-president; Eunice Lewis, Sapulpa, secretary-treasurer.

After the business, the following program was given:

Revision of Course in Elementary Mathematics —R. C. Dragoo

Consumers Mathematics as Taught in Ponca City High School—Anne L. Cowan

Training of Teachers of Mathematics—Professor Stevens, Oklahoma A. & M. College. Paper prepared by Professor Zant.

BE SURE to look at the announcement of the new (the twelfth) *Yearbook* of The National Council of Teachers of Mathematics on the inside back cover of this issue. Read also the description of "Numbers and Numerals" on the back outside cover. One teacher alone has ordered 180 copies of this interesting new monograph and another has ordered 175.

NEW BOOKS

Integrated Mathematics, Intermediate Course, Book III. John A. Swenson, Edwards Brothers, Ann Arbor, Michigan, 1937. Pp. 8+605.

The history of the training of teachers in this country may be divided approximately into two eras: (1) the era in which it sufficed if the teacher knew fairly well the subject to be taught, although he had no special training in its teaching; and (2) the era in which he was trained to teach some branch of knowledge without any adequate knowledge of the subject itself. The first era began in the days of "the little red school house" and extended in the elementary schools through most of the nineteenth century and often for some time beyond. The second era began with some of the normal schools and reached its climax in the years of the present generation. The same swing of the pendulum is seen in all other branches of our intellectual life—in the attitude of the world to social activities, to religion, home life, warfare, government affairs, and so on. The fact that the pendulum swings and swings and swings again is potent not only in the old-time clock but in our dress, our codes of law, our statecraft and our teaching of mathematics as well as other branches of knowledge.

The swing of the pendulum of mathematics has seemingly reached its limit of failure in recent years in our schools of education. In spite of all denials, these schools have tended to insist that it is not necessary to know mathematics beyond the meager offering of the textbooks to be used, provided the candidate for a teacher's diploma has had a prescribed number of "points" in such subjects as the professors of education require. It is a case of either knowing mathematics with an adequate knowledge of the way in which it should be taught, or of pretending to know how to teach a subject about which the teachers have no sufficient information.

It is natural that the advocates of the first of these methods of approach should feel that much time is wasted in the large number of courses in Education X, where X is an unknown quantity and of doubtful value, often conducted by men or women apparently lacking in appreciation of any of the great branches of knowledge. It is equally natural that the narrow-minded leader in Education X should have little sympathy with what he feels to be the narrow-minded teacher of some special branch of learning.

Dr. Swenson feels that the teachers of mathematics should know the subject, but being himself a teacher of rare skill, he feels that mathematics should be taught with a view to its beauty, its strength and wide range of application to all walks of life. He uses the word *integrated* in a better sense than that adopted by those who simply adopt some such catch phrase as we see in "facism," "nazi-ism," "pioneer of learning," "sovietism," "fore-front ranks," "educational field-marshals," and the like; but rather in its primitive significance of "complete" —at least a general view of mathematics as a whole instead of seeing it as a succession of minute photographs on a moving film.

This book begins by offering a general view of number as connected with line, not merely a single straight line but a line having direction in a plane, and so in due time the complex number is made as real as the numbers of childhood. The simpler numbers are at once connected with the simple figures of intuitive geometry, and a natural, interesting, and useful integration is revealed. From this step, Dr. Swenson proceeds to the representation of numbers by letters and to revealing the meaning and usefulness of operations with directed numbers, with powers of numbers, and with the study of monomials, "integrating" all this material and showing the completeness of the view of number in all its elementary phases.

Not to extend this view of Dr. Swenson's plan of teaching and of acquiring useful knowledge, it must suffice to call attention to his sequence of topics to be integrated into one great (for the pupil) whole. He integrates the operations, with monomials, irrationals, polynomials, fractions, complex numbers, equations and their applications, series, elementary functions, geometry, logarithmic and trigonometric functions, and variation. He concludes his work in "integration" by an excellent presentation of the subject of probability and statistics, a branch of algebra of which the value is rapidly becoming appreciated in the schools.

Now to "integrate" the book—taken as a whole it is a very valuable contribution to our educational literature. It is not a textbook for all pupils; indeed it is not so intended. It has been written for the one or two upper grades of the high school, and especially for teachers who know their subject and for pupils who show some ability in elementary mathematics. To require it of the sub-normal pupil, or one who has no liking for the subject, would be to court failure and disappointment. To place copies in the

libraries of the mathematics department (if our philosophers of education and our front-rankers will permit of such a term) or on the shelves of the high school library is essential if education itself is to be "integrated."

The work is well written and well arranged; it has an index which will be appreciated by all users of the rather extended book. A word of praise is also due to the printers for the clear typography used, in spite of the numerous cases in which topics are split in the turning of the pages. This latter, however, is not so important as in the ordinary textbook. Every teacher of mathematics should possess, or at least have access to, the book, for it has a wealth of material which cannot safely be neglected.

DAVID EUGENE SMITH

Elelmentarnyje Priblizhonnyje Vychislenija (Elementary Approximate Computations) by Professor M. L. Frank. 2-d edition. State Technico-Theoretical Publications, Moscow, 1933.

Teoria I Praktika Vychislenij (Theory And Practice Of Computations) Part I, Arithmetic Of Approximate Computations. A text-book for Higher Pedagogical Schools. By V. Bradis. 3-d edition. State Educational and Pedagogical Publications, Moscow, 1933.

Priblizhonnyje Vychislenija (Approximate Computations) by J. Bezikovitch. 4-th edition. State Technico-Theoretical Publications, Moscow, 1933.

The Method of Approximate Computation is not something new. This field is not a new theory; in practice, Approximate Computation should represent a commonly used procedure in every mathematics classroom. This fact was recognized by the National Committee on Mathematical Requirements. That was in 1923. Since then about 88 algebra text-books and 34 trigonometry books were published in this country. Only in two algebra texts pitiful attempts have been made to cope with the problem of Approximate Computation, and according to the knowledge of the writer one of these two texts had a comparatively small circulation (the text was branded as being "too revolutionary and too modern"); the other text, a revision of an older work by a very prominent writer is losing ground. Not in one of the trigonometry text-books do we come across a word about Approximate Computation.

The three Russian books devoted to the topics of Approximate Computation are designed to serve the purpose of presenting these topics on three levels:

1. The elementary level of secondary schools. This is well served by the book of Professor Frank.
2. The Normal School or Teachers College level. For this purpose the book of Bradis is offered.
3. The College or University level. For this purpose the book of Bezikovitch is designed.

It is interesting to note that "The 1933 course of study for the departments of mathematical and physical sciences of the Pedagogical Institute provides for a course in the Theory and Practice of Computations for which 90 hours are assigned" (quoted from the Introduction to the book of Bradis). The work of Bradis is the result of lectures delivered by him for many years in the Tver (now the Kalinin) Pedagogical Institute, and as he tells us, this course is given to Freshman classes so that the students develop the proper technique and acquire the correct ideas necessary for future work. There is also a second part of the book by Bradis.

The first 90 hours of the course in Approximate Computation are devoted to the Arithmetic of this topic. The main purpose of this work is, as stated above, to give the practical phase the more prominent place. Unfortunately, the theoretical phase suffers therefore, and much could have been improved. The theory of errors, the theory of significant digits, and the practical rules based on these theories are analyzed from the general point of view, so that many details that would result in slight modifications of the rules are disregarded. The mathematical knowledge required for the mastery of the book is limited to advanced algebra. The material presented in this book is somewhat condensed, and the students have to make use of their mathematical knowledge to master the contents of the course.

A very fine feature of Bradis' work is that the problem of Approximate Computation is presented from several points of view, or several possible angles:

1. The theory of errors.
2. The theory of significant digits.
3. The theory of the limiting values either of the errors or of the measures.

Much use has been made of the elementary processes of the theory of Least Squares and of elementary Probability Theory, and the rules for the numerical processes derived therefrom are shown to be in general agreement with all the rules derived previously. The writer does not agree with this last procedure because it is dangerous to substitute the mean square (probable) error for the actual apparent error or for the upper limit of the absolute error of a numerical

value of a given measure. The generality of the rules is lost thereby.

Much space is devoted to practical problems, to calculating machines, and to the slide rule. The theory of Mathematical Tables and the theory of linear interpolation, i.e., the interpolation by proportional parts are thoroughly developed. A separate chapter is devoted to formulas for contracted multiplication, contracted division, contracted raising to a power and extraction of roots, to calculations of logarithms and of the exponential function. The book contains numerous illustrative examples as well as a considerable number of problems for solution.

As a text-book for prospective teachers the work of Bradis is a novel thing, and the fact that it is in its third edition (20,000 copies!) proves that it has a wide circulation in Soviet Russia. The book is recommended by the Commissariat of Education and of Public Instruction of the Soviet Union as a text-book for Pedagogical Schools.

Professor Frank's work is designed for those who have had not more than the average course of elementary mathematics. The book covers the four numerical processes as well as the raising to a power and the extraction of roots. Two chapters are devoted to the use of tables, to logarithms, and to the slide rule. The book contains several hundred problems for solution and questions of general character. The purpose of the book is obviously to develop the practical skill of the proper procedure in the numerical processes. The fact that the second edition of this book consisted of 10,000 copies demonstrates the growth of interest in Approximate Computation as a school subject in the Soviet Union.

The book by Bezikovitch is a text for College and University students. It covers the material presented in works on Practical Numerical Analysis such as those by Scarborough or Whittaker and Robinson, but on a more thorough scale, with emphasis on the practical phase of the subject as well. The most interesting part of the book consists in the fact that the first 100 pages are devoted to the theory of the Arithmetic of Approximate Computation. The rules are developed from a strictly rigorous mathematical point of view, but unfortunately no generalization is arrived at. Apparently, the author gave up his attempt to cope with this phase. The book conveys the idea that students taking the course in Approximate Computation in a Russian University are expected not only to master the theory, but to develop adequate skill in solving problems. The book contains many illustrative examples as well as numerous problems for solution.

It is to be regretted that the authors of the three books did not devote a chapter or two to the analysis of the approximativeness of the logarithmic and trigonometric functions in relation to the approximate arguments. Moreover, they would have modified their conclusions concerning the rules for numerical processes if they had thoroughly examined the various processes in detail and realized the complexity of the theory of numerical processes. A recognition of this fact should have resulted in a more comprehensive treatment of the subject.

AARON BAKST

SALE ON YEARBOOKS

We cannot any longer supply the *First Yearbook* which is now out of print. We still have copies of all of the rest of the yearbooks (numbers 2 to 12 inclusive) and will send the entire lot postpaid for $15 which is a saving of 20%.

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